

Problem 1: Grim Trigger

Consider an infinitely repeated game where players play the static Prisoner's Dilemma game in table 1 each period.

		Player 2	
		C	D
Player 1	C	x, x	$0, y$
	D	$y, 0$	$1, 1$

Table 1: The Game

Suppose $1 < x < y$. Players have discount factor, δ . Suppose both players follow a grim trigger strategy. Find the conditions on δ under which the strategies form a Nash Equilibrium and cooperation is sustained in all periods. (I.e., find a condition on the player's patience such that players choose to cooperate in every period.)

How does δ depend on x ? What happens as $x \rightarrow 1$ or $x \rightarrow y$. What is the intuition behind the results?

Problem 2: Repeated Output Game

Consider an industry that consists of two firms, A and B. Each can produce either a high level of output or a low level of output. Firms choose output simultaneously. The payoff matrix of the game is shown in table 2.

1. Suppose the game is to be played only once. Show that high output is a dominant strategy for both players.

		Firm B	
		High	Low
Firm A	High	-2,-2	3,-3
	Low	-3, 3	2,2

Table 2: The Game

2. Now suppose this game is to be played an infinite number of times. Consider the following trigger strategy. A firm chooses low in the first period. In later periods, he chooses low if his opponent has chosen low in every period up to that point; otherwise he chooses high. For what range of values of the discount factor do these trigger strategies constitute a Subgame Perfect Nash Equilibrium?

Problem 3: Advertising

Consider two firms that can either advertise or not advertise. If they both advertise they each earn a profit of 5. If one advertises and the other does not, then the firm that advertises earns 10 and the other firm earns 3. If neither advertises, then each earns an amount α , where $5 < \alpha < 10$. Firms make their decisions on advertising simultaneously.

1. Suppose this game is played once. Show that both firms will advertise.
2. Now suppose this game is to be played an unknown number of times. Both firms know that there is a probability 0.1 that the current play of the game will be the last time the game will be played. So there is a probability 1.0 that the game will be played at least once, a probability 0.9 that the game will be played at least twice, a probability 0.81 that it will be played at least three times, and so on. Consider the following trigger strategy. A firm does not advertise the first time the game is played. In later periods, it does not advertise if its opponent has not advertised in every period up to that point; otherwise it advertises. The discount

factor is 1 (i.e., the interest rate is 0). For what range of values of α do these trigger strategies constitute a Subgame Perfect Nash Equilibrium?

Problem 4: Repeated Bertrand

Consider an industry that consists of two firms, A and B . They face a demand curve $Q = q_A + q_B = 14 - P$, where P is the industry price of output. Both firms have constant marginal cost of \$2.

1. Suppose they form a cartel and choose the price that maximizes the sum of their profits. Show that they will choose $P = \$8$.
2. Now suppose that instead of forming a cartel, they choose prices simultaneously. If they choose different prices, the firm that chooses the lower price captures the entire market; if they set the same price they split the market evenly. Suppose they play this game once. Show that in a Bertrand equilibrium, both firms will charge \$2.
3. Suppose they play this game an infinite number of times. Consider the following grim trigger strategy. Choose the cartel price (i.e., $P = \$8$) in the first period. Continue to choose the cartel price in subsequent periods if all firms have always chose the cartel price up to that point. If any firm chose a price other than \$8, choose the Bertrand price (i.e., \$2) from that point forward. For what range of values of the discount factor do these trigger strategies constitute a subgame perfect equilibrium?
4. Now change this game so that there are $N \geq 2$ oligopolists; thus if they all charge the same price, each will sell a proportion $1/N$ of the market demand at that price. Express the critical discount factor (required to obtain an equilibrium in the infinitely repeated game) as a function of N . Does your answer suggest that it will

be easier to sustain cooperation when N is small or when N is large? What is the intuition behind this result?

Problem 5: Limited Punishment

Consider the game in table 3. The game is repeated infinitely. Both players discount payoff streams using the discount factor $\delta = 0.9$. Players both play limited punishment trigger strategies with a punishment phase of T periods. Determine the length of the punishment period that is required to support $(4, 4)$ as the payoff in every stage of a subgame perfect equilibrium. (I.e., how small can we make T , such that no player finds it optimal to deviate in any period?).

		Player 2		
		t_1	t_2	t_3
Player 1	s_1	1,1	21,0	0,0
	s_2	0,21	4,4	0,1
	s_3	0,0	1,0	-1,-1

Table 3: The Game