Problem Set 5

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Problem 1: Card Game

Each of two players begins by putting a dollar in the pot. Player 1 is dealt a card that is equally likes to be High or Low. She observes the card, but player 2 does not. Player 1 may *see* or *raise*. If she sees, she shows her card to player 2. (Player 2 does not have a card for player 1 to see, but you can imagine her holding a fixed card with a value between High and Low.) If player 1's card is High, she takes the money in the pot, and if it is Low, player 2 takes the money in the pot; in both cases the game ends.

If player 1 raises, she adds $k to the pot and player 2 chooses whether to *pass* (fold) or *meet* (call). If player 2 passes, player 1 takes the money in the pot. If player 2 meets, she adds $k to the pot and player 1 shows her card. If the card is High, player 1 takes the money in the pot, while if it is Low, player 2 does so.

Formulate this situation as an extensive form game of imperfect information. How many information sets does each player have? Find the Nash Equilibrium of this game for $k > 0$. How does player 1's propensity to bluff (raising when her card is Low) depend on the value of k?

Problem 2: NE, SPNE, and PBE

In the extensive-form games in figures 1 and 2, derive the strategic (or normal) form of the games. Then find all pure-strategy Nash, sub-game perfect Nash, and Perfect Bayesian Equilibria.

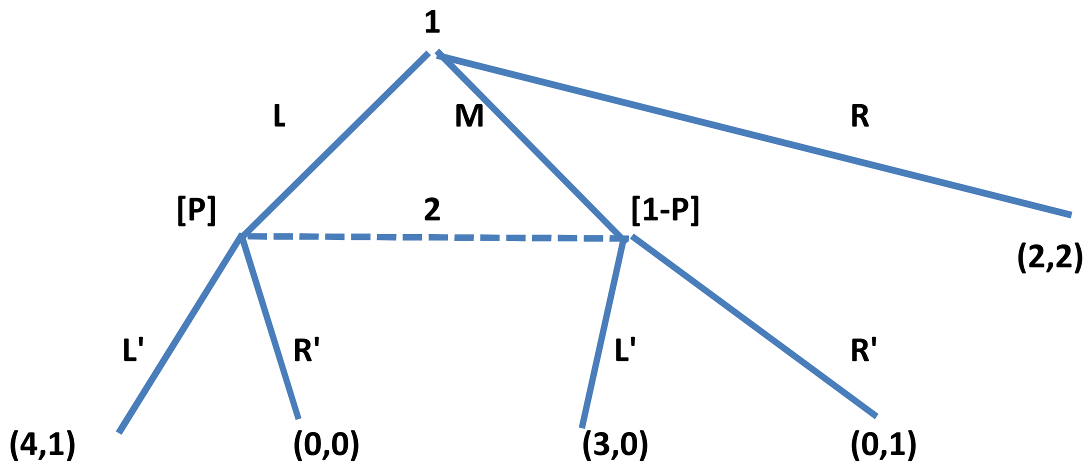

Figure 1: The Game

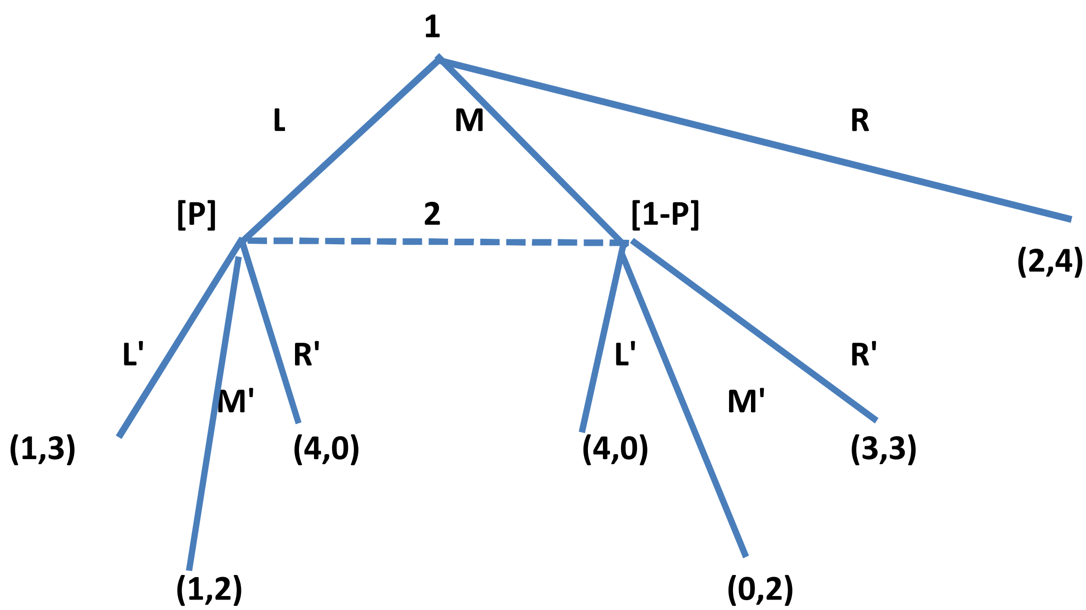

Figure 2: The Game

Problem 3: Signalling

Consider the game in figure 3. Specify a pooling Perfect Bayesian Equilibrium in which both the strong sender and the weak sender play R. Note that nature moves first in this game choosing the sender's type (strong or weak) with equal probability. The sender knows her type, but the receiver does not. Then the sender moves choosing L or R. Then after observing the sender's signal, the receiver moves and chooses u or d.

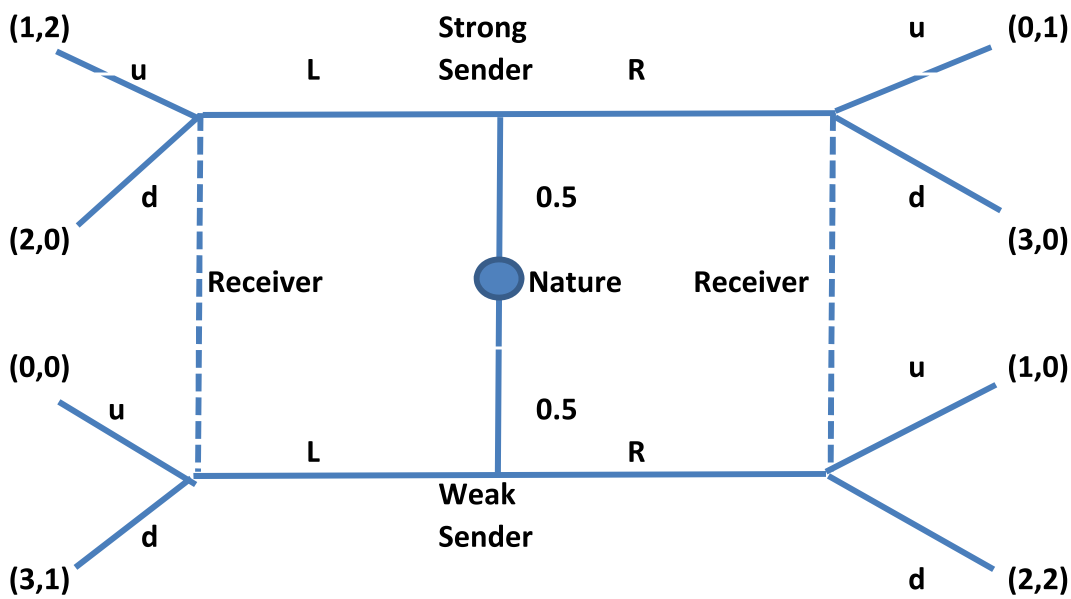

Figure 3: The Game

Problem 4: Lemons

Beverly would like to sell her sports car and Jim would like to buy a sports car. Both know the following:

- Half of all sports cars are lemons and half are good cars.

- If the car is a lemon, it is worth \$2,000 to the buyer and \$1,000 to the seller; if the car is good, it is worth \$6,000 to the buyer and \$4,500 to the seller.
- The seller knows the condition of the car but the buyer does not.

Beverly places an ad in the newspaper and offers a take-it-or-leave-it price of P. Assume throughout this question that if Jim is indifferent between buying and selling he will buy.

1. Show that there is a separating equilibrium in this game where only lemons are sold.
2. Show that the game does not have a pooling equilibrium where all cars are sold.
3. Now consider mixed strategies:
   - If the car is good, Beverly offers to sell it for \$5,000. If the car is a lemon, she offers to sell it for \$5,000 with probability p and offers to sell it for \$2,000 with probability $1 - p$, where $0 < p < 1$.
   - Jim accepts offers of \$5,000 with probability q, where $0 < q < 1$. He always accepts a price of \$2,000.

   Find the values of p and q that yield a Perfect Bayesian equilibrium. [Hint: If the car is good, and Bev offers it for \$5000, then if Jim accepts, the payoffs are (\$5000, \$6000-\$5000 = \$1000) for Bev and Jim respectively. While if Jim rejects, the payoffs are (\$4500, \$0). This is the "assets at the end of the game" approach. Equivalently, we could express the payoffs as the "change in assets" by subtracting \$4500 from each of Bev's payoffs so she gets \$500 if Jim accepts, and \$0 if he rejects.]

Problem 5: Three Types of Equilibria

For each of the games in figure 4, find all pure-strategy:

- Nash Equilibria,
- Subgame Perfect Equilibria,
- Perfect Bayesian Equilbria.

Note the payoffs are listed vertically with player 1's on top, players 2's on bottom. The open circle node in the game in (b) means that if the game reaches that node, nature chooses to end the game with probability $2/3$ and payoffs $(2, 10)$ or continue to player 2's information set with probability $1/3$. Both players are risk neutral.

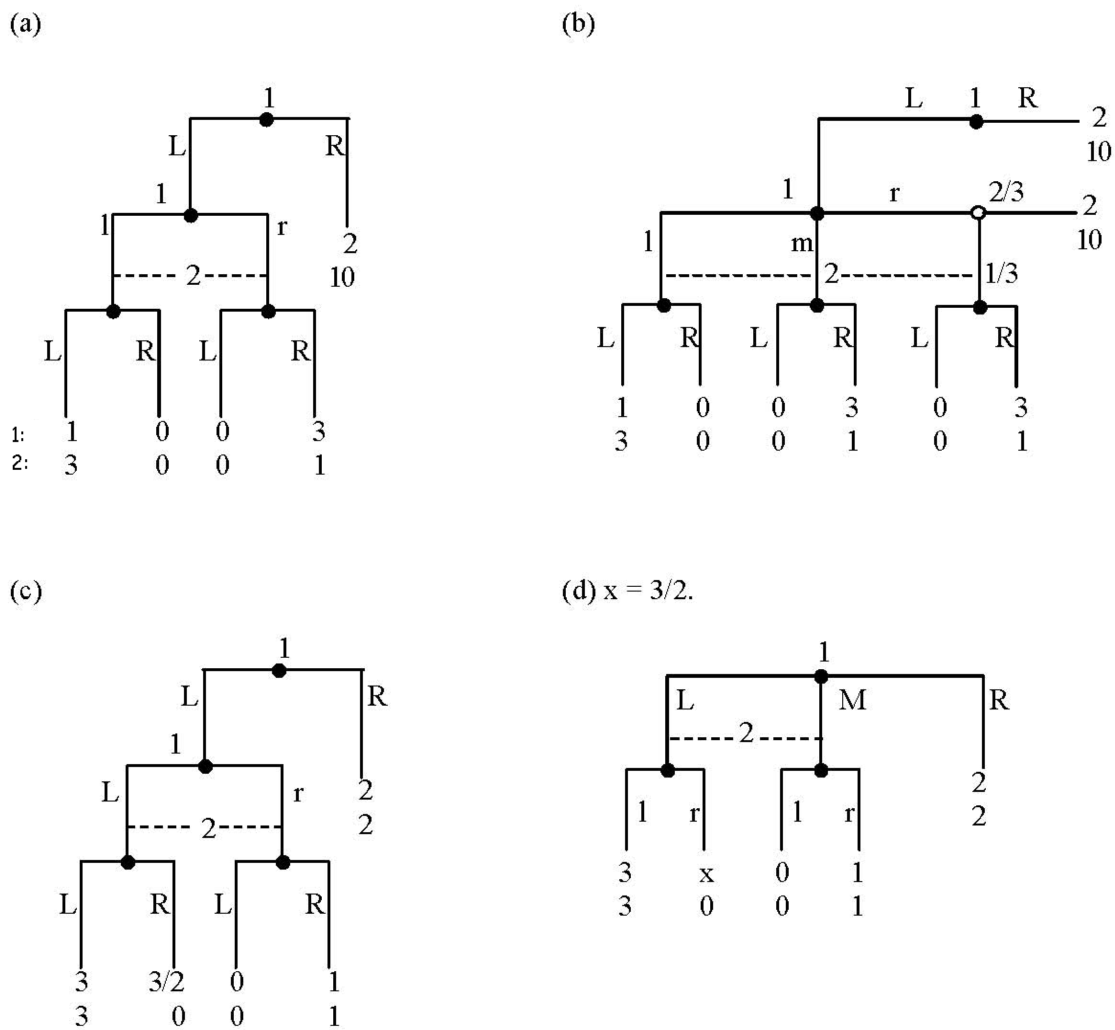

Figure 4: The Games