Problem Set 4

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Problem 1: Bertrand Duopoly Game with Entry

Suppose an incumbent firm faces the possibility of entry by a challenger. The challenger moves first and decides whether to enter and pay entry cost, $f > 0$, or to stay out. If the challenger enters, the incumbent may acquiesce and the two firms split the monopoly profit. If the incumbent fights, they play a static Bertrand game and earn equilibrium profits from that game. Formulate the situation as an extensive form game and find the Sub-game Perfect Nash Equilibria. Remember that an equilibrium is a set of strategies, one for each player, that explicitly states what the players will do at all possible nodes of the game. Assume demand: $P(Q) = \alpha - Q$ and costs: $C(q) = cq$ for the incumbent and $C(q) = cq - f$ for the challenger, with $\alpha > c$.

Problem 2: Smith/Brown

Consider the 2 cases in tables 1 and 2:

		Brown	
		L	R
Smith	T	1,1	0,0
	B	0,0	0,0

Table 1: Case 1

In this game, nature first determines if the payoffs are as in case 1 or as in case 2. Both cases are equally likely. Then Smith learns whether nature has drawn case 1 or case 2, but Brown does not. Smith chooses T or B and Brown simultaneously chooses L

		Brown	
		L	R
Smith	T	0,0	0,0
	B	0,0	2,2

Table 2: Case 2

or R. Find all the pure-strategy Bayesian Nash equililbria of the game.

Problem 3: First-Price Sealed-Bid

Consider a first-price sealed-bid auction in which N bidders' valuations are uniformly distributed between 0 and 1. That is:

$$v_i \;\sim\; U[0,1] \;\forall\; i = 1 \ldots N. \tag{1}$$

Furthermore, assume the v_i are all independent. Show that in a Bayesian Nash equilibrium, each player will bid $(N-1)/N$ times her valuation. [Hints: assume the equilibrium bidding function is linear: $b_i(v_i) = c_i v_i$ where c_i is a constant. Note that $Prob$(Bidder 1 wins) = $Prob(b_1 > b_2, b_1 > b_3, \ldots, b_1 > b_N)$.]

Problem 4: Adverse Selection

Firm A (the "acquiring firm") is considering taking over firm T (the "target firm"). It does not know firm T's value; but knows its value (under T's management) is $x \sim U[0,100]$. That is, the target firm's value is worth between 0 and 100, all values equally likely. Firm T will be worth 50% more under firm A's management that it is worth under its own management. Suppose that firm A bids y to take over firm T. Then if T accepts A's offer, A's payoff is $\frac{3}{2}x - y$ and T's payoff is y. If T rejects A's offer, A's payoff is 0 and T's payoff is x.

Model this situation as a Bayesian game in which firm A chooses how much to offer and firm T decides the lowest offer to accept. Find the Nash Equilibrium (equilibria?) of the game. Explain why the logic behind the equilibrium is called *adverse selection*.

Problem 5: Guess the Average

Three players simultaneously pick a point in the interval $[0, 1]$. The player closest to the average of the three points wins \$1. If there is a tie, then the dollar is split equally among the winners. More formally, the players simultaneously choose strategies s_i from $S_i = [0, 1]$. The average of their choices is $\overline{s} = (s_1 + s_2 + s_3)/3$. Player i's payoff function is:

$$U_i(s_1 + s_2 + s_3) = \begin{cases} 1/t & if\ i \in argmin_j\ |s_j - \overline{s}| \\ 0 & otherwise \end{cases}$$

where t is the number of players who tie (their choices are equally close to the average). (If only one player is closest to the average, then $t = 1$ and that player wins the entire \$1.)

- What are the pure-strategy equilibria of this game?
- What are the mixed-strategy equilibria if the possible strategies are limited to playing 0 or 1, rather than $[0, 1]$?