Problem Set 3

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## Problem 1: Strategies in Extensive Form

- An incumbent faces the possibility of entry by a challenger. (The players may be firms in an industry, politicians vying for a position, or animals competing for a mate.) The challenger may enter or not. If it enters, the incumbent may either acquiesce or fight. What are each of the two players' potential strategies in this game?

- Political figures, Rosa and Ernesto, have to choose either Berlin (B) or Havana (H) as the location for the party congress. They choose sequentially. A third person, Karl, determines who gets to choose first. Both Rosa and Ernesto care only about the actions they choose, not about who chooses first. Rose prefers the outcome in which she and Ernesto choose B to that in which they both choose H, and prefers this outcome to either of the ones in which she and Ernesto choose different actions; she is indifferent between these last two outcomes. Ernesto's preferences only differ from Rosa's in that the roles of B and H are reversed. Karl's preferences are the same as Ernesto's. Model this situation as an extensive form game with perfect information and solve for all pure-strategy Nash Equilibia. [I.e., write down the game tree, list each players' potential strategies, specify payoffs for each outcome according to the preferences, and solve the game.]

## Problem 2: Subgame Perfection

Find the Sub-game Perfect Nash Equilibria (SPNE) of the game in figure 1.

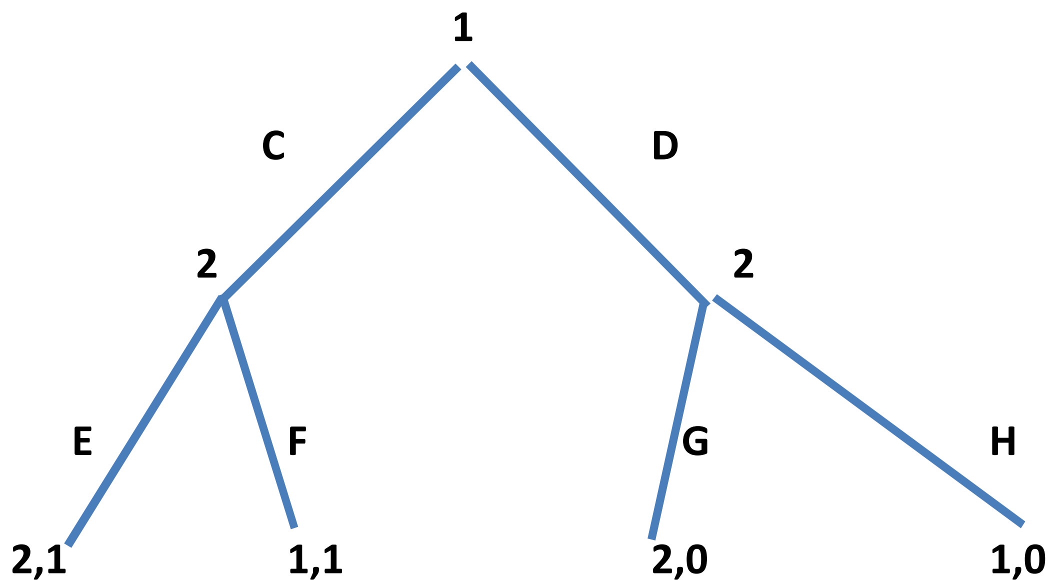


Figure 1: The Game

## Problem 3: Burning Bridges

Army 1, of country 1, must decide whether to attack army 2, of country 2, which is occupying an island between the two countries. In the event of an attack, army 2 may fight, or retreat over a bridge to its mainland. Each army prefers to occupy the island than not to occupy it; a fight is the worst outcome for both armies. Model this situation as an extensive game with perfect information and show that army 2 can increase its subgame perfect equilibrium payoff (and reduce army 1's payoff) by burning the bridge to its mainland (assume this act entails no cost), eliminating its option to retreat if attacked.

## Problem 4: Entry Game

An incumbent firm in an industry faces the possibility of entry by a challenger. First the challenger chooses whether to enter or not. If it does not enter, neither firm has any further action; the incumbent's payoff is $T * M$ (it obtains the monopoly profit, $M$, for each of the following $T \geq 1$ periods) and the challenger's payoff is zero. If the challenger enters, it pays an entry cost, $f > 0$, and in each of the $T$ periods, the incubment first commits to fight or cooperate with the challenger in that period, then the challenger chooses whether to stay in the industry or to exit. (Note the order of the moves in any given period in this game is different from problem 1.) If, in any period, the challenger stays in, each firm obtains, in that period, the profit of $-F < 0$ if the incumbent fights and $C > Max\{F, f\}$ if the incumbent cooperates. If, in any period, the challenger exits (it cannot re-enter), both firms obtain the profit of zero in that period (regardless of the incumbent's action); and in subsequent periods, the incumbent obtains $M > 2C$ and the challenger receives zero. Each firm only cares about the *sum* of its profits over all periods.

Find the (pure-strategy) SPNE of the extensive game that models this situation.

## Problem 5: Dollar Auction

An object that two people value at an integer value, $v > 0$, is sold in an auction. In the auction, the people take turns bidding; a bid must be a positive integer greater than the previous bid. (In the situation that gives the game its name, $v$ is 100 cents.) On her turn, a player may pass rather than bid, in which case the game ends and the other player receives the object; *both* players pay their last bid (if any). (For example, if player 1 initially passes, player 2 receives the object at no cost; if player 1 bids 1, player 2 bids 3, and then player 1 passes, player 2 obtains the object, but player 1 pays 1 and player 2 pays 3.) Each person's wealth is $w > v$ and players may not bid more than their wealth.

Model the auction as an extensive game with $v = 2$ and $w = 3$. Find its (pure-strategy) subgame perfect Nash Equilibria. Think about how one would solve this game for arbitrary values of $v$ and $w$ (but do not attempt to solve it unless you want to!).

## Problem 6: The Rotten Kid

A child's action, $a$ (a number), affects both her own private income, $c(a)$, and her parents' income, $p(a)$. For all values of $a$, $p(a) > c(a)$. The child is selfish: she only cares about the amount of money she has. Her loving parents care both about how much much they have and how much their child has. Specifically, model the parents as a single player whose preferences are represented by a payoff equal to the smaller of the amount of money they have and the amount of money their child has. The parents may transfer money to their child. First the child takes an action ($a$), and then the parents decide how much money to transfer ($t$) to the child.

Model this situation as an extensive game and show that in a subgame perfect equilibrium, the child takes an action that maximizes the sum of her private income and her parents' income. (In particular, the child's action does not maximize her own private income.)