Problem Set 2
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Problem 1: Second-Price Sealed-Bid

Assume there are n bidders indexed by $i = 1 \ldots n$. Assume the following valuations of the bidders:

$$v_1 > v_2 > \cdots > v_n. \quad (1)$$

So bidder n has the *lowest* valuation of all the bidders. Find a Nash Equilibrium of a second-price sealed-bid auction in which player n obtains the object.

Problem 2: Mixed Strategy Nash Equilibria

Find *all* Nash Equilibria of the strategic games in tables 1 and 2.

		Player 2: L	Player 2: R
Player 1	T	6,0	0,6
	B	3,2	6,0

Table 1: The Game

		Player 2: L	Player 2: R
Player 1	T	0,1	0,2
	B	2,2	0,1

Table 2: The Game

Problem 3: A Coordination Game

Two people can perform a task if, and only if, they both exert effort. They are both better off if they both exert effort and perform the task than if neither exerts effort (and nothing is accomplished); the worst outcome for each person is that she exerts effort and the other person does not (in which case again nothing is accomplished). Specifically, the players' preferences are represented by the expected value of the payoff functions in table 3, where c is a positive number less than 1 that can be interpreted as the cost of exerting effort. Find all the mixed strategy Nash Equilibria of this game. How do the equilibria change as c increases? Explain the reasons for the changes.

		Player 2	
		No Effort	Effort
Player 1	No Effort	0,0	0,-c
	Effort	-c,0	1-c,1-c

Table 3: The Game

Problem 4: Defending Territory

General A is defending territory accessible by two mountain passes against an attack by general B. General A has 3 divisions at her disposal, and general B has 2. Each general allocates her divisions between the two passes. General A wins the battle at a pass if and only if she assigns at least as many divisions to the pass as does General B; she successfully defends her territory if and only if she wins the battle at both passes. Formulate this situation as a strategic game and find *all* its mixed strategy Nash Equilbia. [Hint: First argue that in every equilibrium, General B assigns probability zero to the action of allocating one division to each pass. Then argue that in any equilibrium, General B assigns probability $\frac{1}{2}$ to each of her other actions. Finally, find General A's equilibrium strategies.]

In an equilibrium, do the generals concentrate all their forces at one pass, or spread them out?

Problem 5: Eliminating Dominated Strategies

Find all mixed strategy Nash Equilibria of the game in table 4 by first eliminating any strictly dominated actions and then constructing the players' best response functions.

		Player 2		
		L	M	R
Player 1	T	2,2	0,3	1,2
	B	3,1	1,0	0,2

Table 4: The Game

Problem 6: Three Strategies

Find all Nash Equilibria of the game in table 5.

		Player 2		
		L	M	R
Player 1	T	2,2	0,3	1,3
	B	3,2	1,1	0,2

Table 5: The Game