Problem Set 1

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Problem 1: IESDS

In the two-player game in table 1, what strategies survive iterated elimination of strictly-dominated strategies? What are the pure-strategy Nash Equilibria?

		Player 2		
		L	C	R
	T	1,3	5,4	4,2
Player 1	M	2,3	3,1	3,2
	B	3,5	4,7	1,4

Table 1: The Game

Problem 2: Hermaphroditic fish

Members of some species of hermaphroditic fish choose, in each mating encounter, whether to play the role of a male or a female. Each fish has a preferred role which uses up fewer resources and hence allows more future mating. A fish obtains a payoff of H if it mates in its preferred role and L if it mates in the other role, where $H > L$. (Payoffs are measured in terms of number of offspring, which fish have evolved to maximize.) Consider an encounter between two fish whose preferred roles are the same. Each fish has two possible actions: mate in either role or insist on its preferred role. If both fish offer to mate in either role, the roles are assigned randomly, and each fish's payoff is $0.5(H + L)$ (the average of H and L). If each fish insists on its preferred role, the fish do not mate; each goes off in search of another partner, and obtains the payoff S. The higher the chance of meeting another partner, the larger is S.

Formulate this situation as a strategic game and determine the range of values of S, for any given values of H and L, for which the game differs from the Prisoner's Dilemma only in the names of the actions.

Problem 3: Prisoner's Dilemma

Consider the following prisoner's dilemma with altruistic preferences. Each of two players has two possible actions, Quiet and Fink; each action pair results in the players receiving amounts of money equal to the numbers corresponding to that action pair in table 2 below (for example, if player 1 chooses Quiet and player 2 chooses Fink, then player 1 receives nothing, whereas player 2 receives 3). The players are not "selfish"; rather, the preferences of each player i are represented by the payoff function $m_i(a) + b * m_j(a)$, where $m_i(a)$ is the amount of money received by player i when the action profile is a, j is the other player, and b is a given nonnegative number. Player 1's payoff to the action pair (Quiet, Quiet), for example, is $2 + 2b$.

- Formulate a strategic game that models this situation in the case $b = 1$. Is this game the Prisoner's Dilemma?

- Find the range of values of b for which the resulting game is the Prisoners Dilemma. For values of b for which the game is not the Prisoners Dilemma, find the Nash equilibria.

Problem 4: A Joint Project

Two people are engaged in a joint project. If each person i puts in the effort x_i, a nonnegative number equal to at most 1, which costs her $c(x_i)$, the outcome of the project

		Player 2	
		Quiet	Fink
Player 1	Quiet	2,2	0,3
	Fink	3,0	1,1

Table 2: The Game

is worth $f(x_1, x_2)$. The worth of the project is split equally between the two people, regardless of their effort levels. Formulate this situation as a strategic game. Find the Nash equilibria of the game in each of the following two cases:

- $f(x_1, x_2) = 3x_1x_2$ and $c(x_i) = x_i^2$ for $i = 1, 2$, and
- $f(x_1, x_2) = 4x_1x_2$ and $c(x_i) = x_i$ for $i = 1, 2$.

In each case, is there a pair of effort levels that yields higher payoffs for both players than do the Nash equilibrium effort levels?

Problem 5: Cournot game

Find the Nash equilibria of a Cournot game when there are two firms, the inverse demand function is given by equation 1, and the cost function of each firm is given by equation 2.

$$\begin{aligned} P(Q) &= \alpha - Q \text{ if } Q \leq \alpha \\ P(Q) &= 0 \text{ if } Q > \alpha \end{aligned} \tag{1}$$

where $\alpha > 0$.

$$\begin{aligned} C_i(q_i) &= 0 \text{ if } q_i = 0 \\ C_i(q_i) &= f + cq_i \text{ if } q_i > 0 \end{aligned} \tag{2}$$

where $c \geq 0$, $f > 0$, and $c < \alpha$. (Note that the fixed cost f affects the firm's decision of whether to operate or not; it does not affect the output a firm wishes to produce, given that it chooses to operate.)

Problem 6: Bertrand game

Consider the variant of Bertrand's duopoly game that we have seen in class. Demand is $Q(p) = \alpha - p$ and each firm is restricted to choose a price that is an integral number of cents. Take the monetary unit to be cent, assume that c is an integer, and $\alpha > c + 1$. Is (c, c) a Nash equilibrium of this game? If they exist, find all other Nash equilibria.