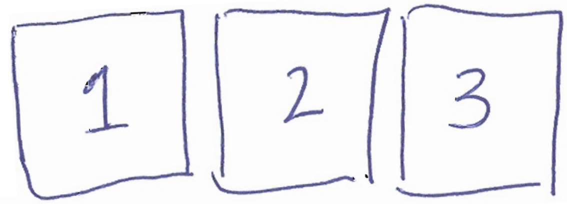
Monte Hall Problem



- 3 Doors.
- Prize Behind One Door
- Contestant Picks Door, Monte opens Another Door with no prize, Contestant can Stick with initial Choice or switch Doors.

→ Should the Contestant Stick or Switch?

- Suppose, WLOG, Contestant picks door 1.
- Let A_i be the event that the prize is behind door i.
- Let B_i be the event that Monte opens door i.

So

$$Pr(B_2) = \underbrace{Pr(B_2|A_1)}_{1/2} \cdot \underbrace{P(A_1)}_{1/3} + \underbrace{Pr(B_2|A_2)}_{0} \cdot \underbrace{Pr(A_2)}_{1/3} + \underbrace{Pr(B_2|A_3)}_{1} \cdot \underbrace{Pr(A_3)}_{1/3}$$

$$= 1/6 + 0 + 1/3 = 1/2$$

[$Pr(B_3) = 1/2$ by Symmetry]

So, Suppose B_2:

$$Pr(A_1|B_2) = \frac{Pr(B_2|A_1)\cdot Pr(A_1)}{Pr(B_2)} = \frac{\frac{1}{2}\cdot\frac{1}{3}}{\frac{1}{2}} = \frac{1}{3}$$

$$Pr(A_2|B_2) = \frac{P(B_2|A_2)\cdot P(A_2)}{P(B_2)} = \frac{0\cdot\frac{1}{3}}{\frac{1}{2}} = 0$$

$$Pr(A_3|B_2) = \frac{P(B_2|A_3)\cdot P(A_3)}{P(B_2)} = \frac{1\cdot\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

So the contestant wins with probability $1/3$ by sticking and wins with probability $2/3$ by switching.