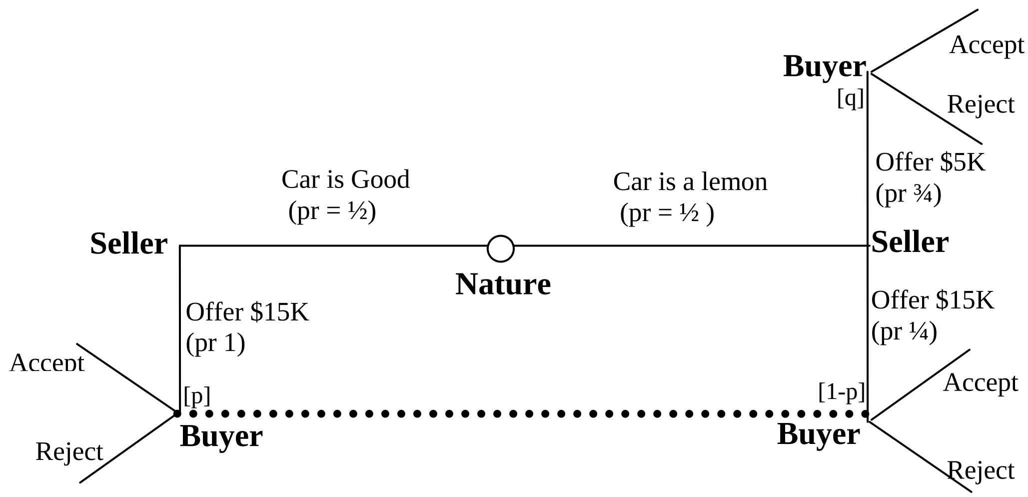
Bayes Rule Example – The "Lemons Problem"

$$Pr(A|B) = P(A,B) / P(B)$$



Ex-Ante, the seller knows the true quality of the car (if it's good or a lemon). The buyer only know the distribution of lemons in the entire population of used cars (½ of all cars are good and ½ are lemons).

Ex-Post, the buyer should update his beliefs using Bayes rule. We need to find the p and the q in the above extensive game with imperfect information.

First off, we know the seller does not offer good cars for $5,000. Hence if the Buyer sees a price of $5,000 he should be certain (pr = 1) that the car is a lemon. Comfirm with Bayes rule:

$q = Pr(\text{Car is a lemon} \mid \text{Price} = \$5000) = Pr(\text{Lemon},\$5000) / Pr(\$5000) = \frac{1}{2} \cdot \frac{3}{4} / (\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot \frac{3}{4}) = 1$

Of course $1\text{-}q = 0$ is the probability that the car is good if the price is $5000 which never happens:

$1-q = Pr(\text{Car is good} \mid \text{Price} = \$5000) = Pr(\text{Good},\$5000) / Pr(\$5000) = 0 / (\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot \frac{3}{4}) = 0$

What if the price is $15000? By Bayes rule:

$p = Pr(\text{Car is good} \mid \text{Price} = \$15000) = Pr(\text{Good},\$15000) / Pr(\$15000) = \frac{1}{2} \cdot 1 / (\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{1}{4}) = 4/5$

And finally,

$1-p = Pr(\text{Car is a lemon} \mid \text{Price} = \$15000) = Pr(\text{Lemon},\$15000) / Pr(\$15000) = \frac{1}{2} \cdot \frac{1}{4} /(\frac{1}{2} \cdot 1+\frac{1}{2} \cdot \frac{1}{4}) = 1/5$

So given the seller's probabilities of each offer and the initial distribution of cars in the population, these p's and q's are the buyers updated (ex-post) beliefs about the quality of the car for sale.