Infinitely Repeated Cournot.

Assume

- $n = 2$
- $Q = \alpha - P \iff P = \alpha - Q$
- $C_i(q_i) = c q_i \quad \forall i$
- $q_1 + q_2 = Q$

Stage Game (G)

- Each Firm Solves

$$\max_{q_i} \left\{ q_i \left[\alpha - q_i - q_j - c \right] \right\}$$

$$\text{FOC} = 0 \implies q_i(q_j) = \frac{1}{2}(\alpha - q_j - c)$$

(Best Response Function)

Solve Simultaneously:

$$q_i^c = q_j^c = \tfrac{1}{3}(\alpha - c)$$

$$\implies \pi_i^c = \tfrac{1}{3}(\alpha - c)\left[\alpha - \tfrac{2}{3}(\alpha - c) - c\right] = \tfrac{1}{9}(\alpha - c)^2$$

Monopolist Problem

$$\max_{Q} \left\{ Q \left[\alpha - Q - c \right] \right\}$$

$$\implies Q^M = \tfrac{1}{2}(\alpha - c)$$

$$\implies \pi^M = \tfrac{1}{4}(\alpha - c)^2$$

- CONSIDER $G(\infty, \delta)$ AND TRIGGER STRATEGIES

$$\sigma_i = \sigma_j = \begin{cases} \bullet \text{ PLAY } q_i = \tfrac{1}{2}Q^M \text{ IN THE FIRST PERIOD AND AS LONG AS ALL FIRMS HAVE PLAYED } q_i = \tfrac{1}{2}Q^M \text{ IN THE PAST} \\ \bullet \text{ PLAY } q_i^c = \tfrac{1}{3}(\alpha - c) \text{ OTHERWISE (FOREVER!)} \end{cases}$$

EQUILIBRIUM PATH PAYOFFS

$$\pi^e = \tfrac{1}{2}\pi^M(1 + \delta + \delta^2 + \dots) = \frac{\pi^M}{2(1-\delta)}$$

$$= \frac{(\alpha - c)^2}{8(1-\delta)}$$

OPTIMAL DEVIATION?

- USE BEST RESPONSE FUNCTION.

$$q_i^d = \frac{1}{2}(\alpha - \underbrace{\tfrac{1}{2}Q^M}_{q_j} - c) = \frac{1}{2}(\alpha - \tfrac{1}{4}(\alpha - c) - c)$$

$$= \frac{3}{8}(\alpha - c)$$

$\Rightarrow$ ONE PERIOD profits of

$$\tfrac{3}{8}(\alpha - c)\left[\alpha - \tfrac{3}{8}(\alpha - c) - \tfrac{1}{4}(\alpha - c) - c\right]$$

$$= \frac{9}{64}(\alpha - c)^2$$

$$\pi^d = \frac{9}{64}(\alpha - c)^2 + \frac{1}{9}(\alpha - c)^2 \Big[\underbrace{\delta + \delta^2 + \delta^3 + \dots}_{\delta/1-\delta}\Big]$$

So Cooperation (ie joint monopoly) is optimal if

$$\Pi^e \geq \Pi^d$$

$$\frac{(\alpha-c)^2}{8(1-\delta)} \geq \frac{9}{64}(\alpha-c)^2 + \frac{1}{9}(\alpha-c)^2\left(\frac{\delta}{1-\delta}\right)$$

$$\frac{1}{8(1-\delta)} \geq \frac{9}{64} + \frac{\delta}{9(1-\delta)}$$

$$\frac{1}{8} \geq \frac{9}{64} - \frac{9}{64}\delta + \frac{1}{9}\delta$$

$$\delta\left(\frac{9}{64} - \frac{1}{9}\right) \geq \frac{1}{64}$$

$$\boxed{\delta^* \geq 9/17}$$