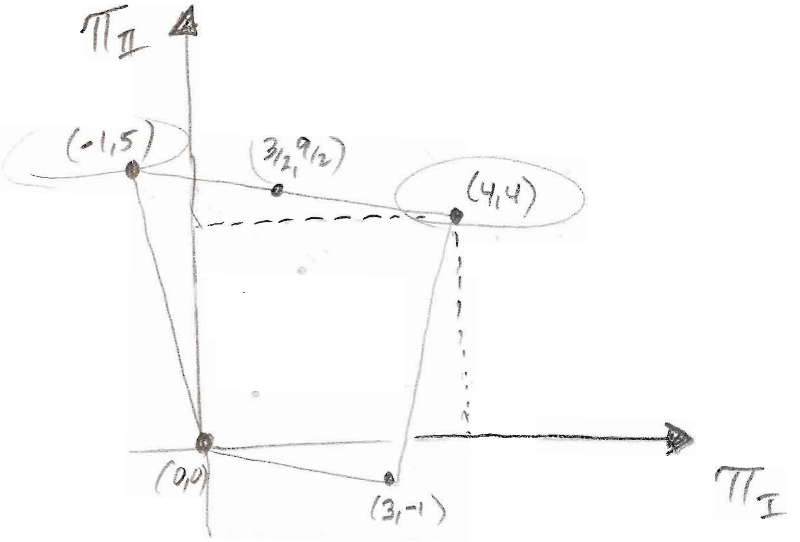
Another Folk Example

		II	
		C	D
I	C	4, 4	−1, 5
	D	3, −1	0, 0



Strategies

$\sigma_I = \begin{cases} \text{Play C in first period and in all subsequent if no deviation has occured} \\ \text{Play D else.} \end{cases}$

$\sigma_{II} = \begin{cases} \text{Play C in odd periods, D in even if no deviation has occured} \\ \text{Play D else} \end{cases}$

$\left[\begin{array}{l} \tfrac12(-1) + \tfrac12(4) = 3/2 \\ \tfrac12(5) + \tfrac12(4) = 9/2 \end{array}\right]$

$$\pi_1^e = 4 - 1\delta + 4\delta^2 - 1\delta^3 + 4\delta^4 - 1\delta^5 \ldots$$
$$= 4(1+\delta^2+\delta^4+\ldots) + (-1)(\delta+\delta^3+\delta^5+\ldots)$$
$$= \frac{4}{1-\delta^2} - 1\left(\frac{\delta}{1-\delta^2}\right)$$

$$\pi_2^e = 4 + 5\delta + 4\delta^2 + 5\delta^3 + \ldots$$
$$= 4/_{1-\delta^2} + 5\delta/_{1-\delta^2}$$

π_1^d [Even Period Dev!] : $4 + 0\delta + 0\delta^2 + 0\delta^3 + \ldots = 4$

π_2^d [Odd Period Dev!] : $5 + 0\delta + 0\delta^2 + \ldots = 5$

So:

$\pi_1^e \geq \pi_1^d$

$$\frac{4}{1-\delta^2} - \frac{\delta}{1-\delta^2} \geq 4$$
$$4 - \delta \geq 4 - 4\delta^2$$
$$-1 \geq -4\delta$$
$$1 \leq 4\delta$$
$$1/4 \leq \delta$$

$\pi_2^e \geq \pi_2^d$

$$\frac{4}{1-\delta^2} + \frac{5\delta}{1-\delta^2} \geq 5$$
$$4 + 5\delta \geq 5 - 5\delta^2$$
$$5\delta + 5\delta^2 \geq 1$$
$$\delta + \delta^2 \geq 1/5$$
$$\delta \geq 0.17$$

$\boxed{\text{So we require } \delta^* \geq 1/4}$