Summer 2010 – Chesnes
90 Minutes

Economics 644 – Midterm Solutions

1. 3 Oligopolists
   a. In a Cournot game, firms choose quantity to maximize their profits so each firm has the same strategy space, $q_i \varepsilon [0,\infty)$.
   b. Since the marginal cost for all firms is constant and equal to six, there are an infinite number of NE of the form:

      $\{P_i = 6, P_j >= 6, P_k >= 6$, j and/or k with equality.$\}$

      Firms all obtain zero profits as they set price equal to marginal cost or they price above the lowest price competitor and have no share of the demand. Note we need at least two players pricing at marginal cost (=6) in order to restrain the firms at cost. If any one firm that is setting P = 6 raised their price above 6, they would lose their share of the market and (still) obtain no profits. If a firm priced below 6, they would win the entire market but sell each unit at a loss, so profits of the deviating firm would be negative. At any set of prices below 6, each firm is either making a negative profit (and should deviate to at least P=6), or is not winning any market share so is making zero profit. At prices above P = 6 (since prices are continuous), a relatively high price firm can always undercut the lowest priced firm by epsilon and win the entire market and make a positive profit. Therefore no NE exists of this type.
   c. In a finite dynamic game, we solve via backward induction. So firm three solves:

      Max(q3) { q3*(33 – (q1+q2+q3)^2) – 6*q3 }

      This will yield a best response function for firm 3 of the form q3(q1,q2). Firm two solves:

      Max(q2) { q2*(33 – (q1+q2+q3(q1,q2))^2) – 6*q2 }

      This will yield a best response function for firm 2 of the form q2(q1). Firm one solves:

      Max(q1) { q1*(33 – (q1+q2(q1)+q3(q1,q2))^2) – 6*q1 }

      This would yield an optimal q1*, which could then substituted in to q2(q1*) = q2* and then q3(q1*,q2*) = q3*. The SPNE would then be at (q1*,q2*,q3*).
   d. If the firms collude, they will simply split the monopoly profits equally. The monopolist's problem is:

      Max(Q) {Q*(33-Q^2) – 6Q }
      FOC(Q): 33 – 3Q^2 – 6 = 0 → 3Q^2 = 27 → Q* = 3
      P* = 33 – Q*^2 = 33 – 9 = $24
      Monopoly profit = 3*(24-6) = 3*18 = $54

      So firms will each produce Q*/3 = 1 unit, and at a market price of $24, make $18 in profits.

2. Simultaneous move game
   a. See lecture notes.
   b. L and C both dominate R for player 2. The payoffs from player L (or C) are all strictly higher than playing R for ANY action of player 1. Removing R from consideration, M dominates B for player 1. The payoffs from playing M are strictly greater than those from playing B for both of player 2's remaining actions (L and C). So T, M, L, and C all survive IESDS.
   c. The underlining method gives 2 NE at (T,C) & (M,L), or in mixed strategy notation, at {[(1,0,0), (0,1,0)] and [(0,1,0),(1,0,0)]}. There are more mixed strategy NE which can be found by writing down the unconditional expected payoffs for each player assuming that player 1 mixes on (T,M) with probabilities (p,1-p) respectively, and player 2 mixes on (L,C) with probabilities (q,1-q) respectively. So player 1's unconditional expected payoff is:

      $$E[U1] = 5(1-p)*q = 5q - 5pq$$

      Since p, the choice variable for player 1, enters the second term, player 1 will want to set $p = 0$ if $q > 0$ (to make the second term as big as possible – ie not negative) and set $p \varepsilon [0,1]$ if $q = 0$.

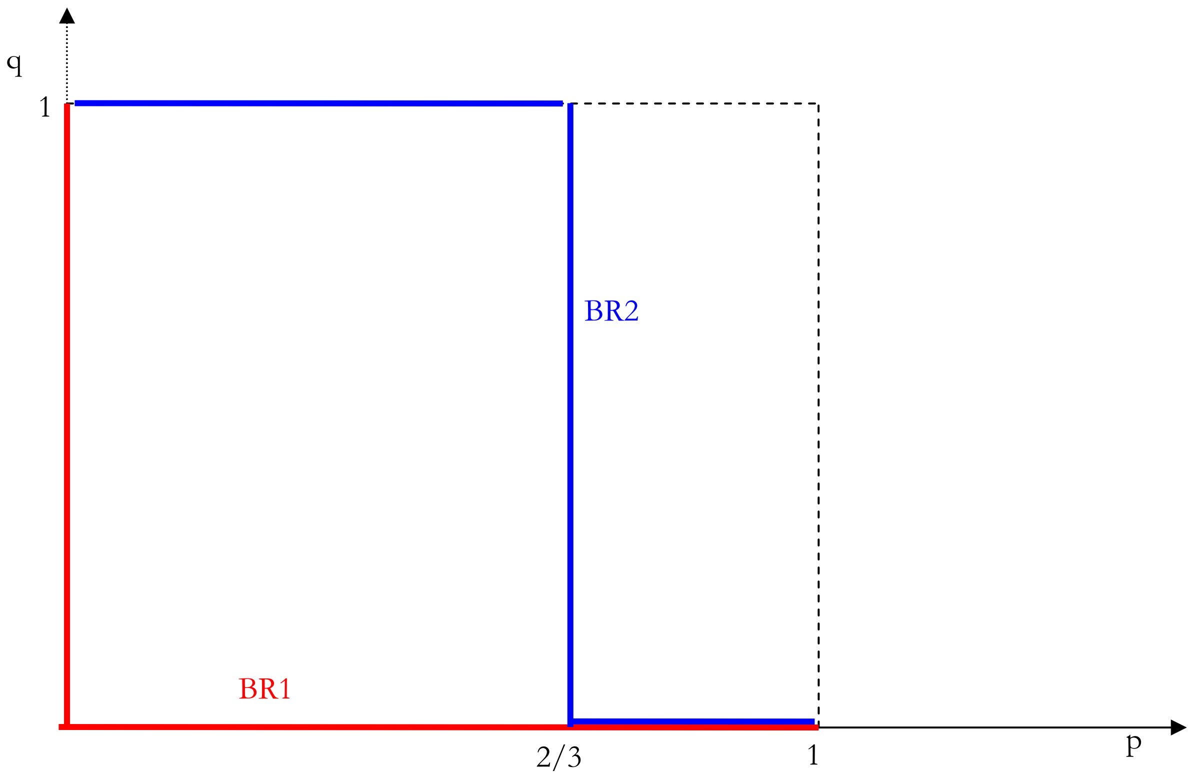
      Player 2's unconditional expected payoff is:

      $$\begin{aligned} E[U2] &= pq + 2(1-p)q + 2p(1-q) \\ &= pq + 2q - 2pq + 2p - 2pq \\ &= -3pq + 2p + 2q \\ &= q(2 - 3p) + 2p \end{aligned}$$

      Since q, the choice variable for player 2 multiplies (2-3p),

      $$\text{If } (2-3p) > 0 \Leftrightarrow p < 2/3 \rightarrow q = 1$$
      $$\text{If } (2-3p) < 0 \Leftrightarrow p > 2/3 \rightarrow q = 0$$
      $$\text{If } (2-3p) = 0 \Leftrightarrow p = 2/3 \rightarrow q \varepsilon [0,1]$$

Graphically,



So ALL the NE are:
{[(0,1,0), (1,0,0)]   and   [(p,1-p,0), (0, 1 , 0) | p >= 2/3] }

Note that the other pure strategy NE we noted above is subsumed in the second NE listed here.

3. Extensive form game
   a. The game has 4 subgames.
   b. For player 1: {AX, AY, BX, and BY}. And for player 2; {CE,CF,DE,DF}.
   c. Consider the following game in strategic/normal form:

Player 2

Player 1		CE	CF	DE	DF
	AX	1,1	1,1	4,0	4,0
	AY	1,1	1,1	4,2	4,2
	BX	1,4	2,2	1,4	2,2
	BY	1,4	2,2	1,4	2,2

So the game has 5 pure strategy NE at: {(AX,CE), (BX,CE), (BY,CE), (AY,DE), (AY,DF)}

   d. Note that player 2 always plays E at the right side node and plays D only if player 1 is playing Y at the bottom node. So, consider the only two strategies of player 2 that have the potential to be part of a SPNE: (CE) and (DE).

Player 2 Potential Strategy	Player 1 Best Response	Player 2 Best Response	Equilibrium? (Column 1 = Column 3?)
CE	AX	CE	Yes
CE	AY	DE	No
CE	BX	CE	Yes
CE	BY	DE	No
DE	AX	CE	No
DE	AY	DE	Yes

So the only SPNE are: { (AX, CE), (BX, CE), and (AY, DE) }