

Midterm Summer 2007 Solutions

1. (40%)

- a. See lecture notes. A player's strategy is strictly dominated by another if it is strictly worse than another strategy for all possible actions of the other player.
- b. T,B,L,R all survive. M and C are dominated.
- c. By the underlining method, we see two pure strategy NE:

$$[(1,0,0), (1,0,0)]$$

$$[(0,0,1), (0,0,1)]$$

Suppose player 1 plays T with probability p and player 2 plays L with probability q . Then:

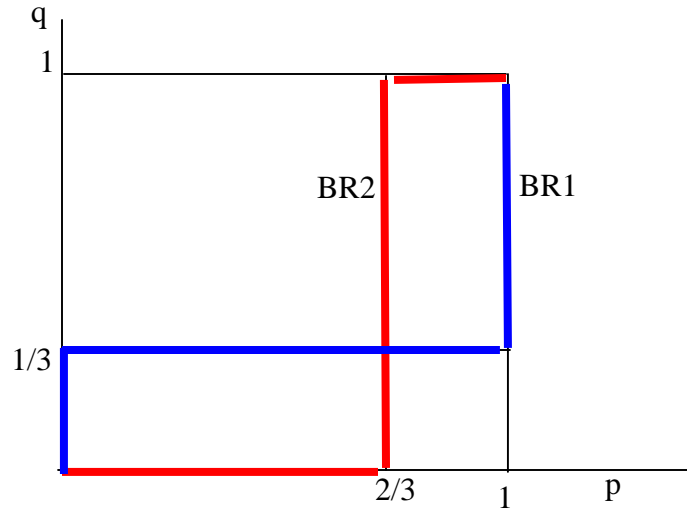
$$E[\text{Payoff1}] = 2pq + 1(1-p)(1-q) = 3pq - p - q + 1 = p(3q - 1) - q + 1$$

$$E[\text{Payoff2}] = 1pq + 2(1-p)(1-q) = 3pq - 2p - 2q + 2 = q(3p - 2) - 2p + 2$$

So there exists a MSNE at:

$$[(2/3, 0, 1/3), (1/3, 0, 2/3)]$$

d.



2. (30%)

- a. Strategies of Player 1: (NA, NB, MA, MB). Strategies of Player 2: (SH, SJ, TH, TJ).
 b. Strategic / Normal Form:

	SH	SJ	TH	TJ
NA	<u>1,2</u>	<u>1,2</u>	<u>3,1</u>	<u>3,1</u>
NB	<u>1,2</u>	<u>1,2</u>	<u>3,3</u>	<u>3,3</u>
MA	<u>1,4</u>	<u>2,2</u>	<u>1,4</u>	<u>2,2</u>
MB	<u>1,4</u>	<u>2,2</u>	<u>1,4</u>	<u>2,2</u>

NE : (NA,SH), (MA, SH), (MB, SH), (NB, TH), (NB, TJ)

- c. Note that player 2 always plays H at the right side node and plays T only if player 1 is playing B at the bottom node. So, consider the only two strategies of player 2 that have the potential to be part of a SPNE: (SH) and (TH).

Player 2	Player 1	Player 2	Equilibrium?
SH	NA	SH	Yes
	NB	TH	No
	MA	SH	Yes
	MB	TH	No
TH	NA	SH	No
	NB	TH	Yes

SPNE: (NA,SH), (MA,SH), (NB,TH)

3. (30%)

- a. Cournot – simultaneous quantity choice.

Firm 1 solves:

$$\text{Max}(q_1) \{q_1(8-q_1-q_2-2)\}$$

$$\text{FOC}(q_1): 8-2q_1-q_2-2=0$$

$$\text{BR}_1 : q_1 = 3-(1/2)q_2$$

Firm 2 solves:

$$\text{Max}(q_2) \{q_2(8-q_1-q_2-4)\}$$

$$\text{FOC}(q_2): 8-q_1-2q_2-4=0$$

$$\text{BR}_2 : q_2 = 2-(1/2)q_1$$

Solve BR's simultaneously yields NE at $(q_1^*, q_2^*) = (8/3, 2/3)$.

- b. Stackelberg – dynamic quantity choice. Note the firm 2's BR function is as in part (a). Thus:

Firm 1 solves:

$$\text{Max}(q_1) \{q_1(8-q_1-(2-(1/2)q_1)-2)\}$$

$$\text{Max}(q_1) \{q_1(4-(1/2)q_1)\}$$

$$\text{FOC}(q_1): 4-q_1=0 \rightarrow q_1 = 4$$

Plug into the BR of firm 2, $q_2 = 0$. SPNE is thus $(q_1^*, q_2^*) = (4, 0)$.