Spring 2008 Midterm Solutions

1. ***(50%) Nash Equilibria in Pure and Mixed Strategies.***

   a. **See Osborne for a definition. Essentially the payoff from the NE strategies must yield a *strictly* higher payoff than what each player could get by *unilaterally* deviating.**

   b. **If player 2 plays X, the mixture of (0, 0.5, 0.5) dominates A. If player 2 plays Y, the mixture (0, 0.5, 0.5) yields an expected payoff of ½. So in order to dominate A, G < ½. Note this is a STRICT inequality.**

   c. **If G is zero, it satisfies the inequality in (b) so we can eliminate A from consideration. By the underlining method, we have 2 PSNEs at (C,X) and (B,Y).**

If player 1 mixes on (0,p,1-p) and player 2 mixes on (q,1-q), we have:

$$\text{Max }\{E[U_1] = 1pq + p(1-q) + 3(1-p)q\}$$
$$\text{Max }\{E[U_1] = pq + p - pq + 3q - 3pq\}$$
$$\text{Max }\{E[U_1] = p(1-3q) + 3q\}$$

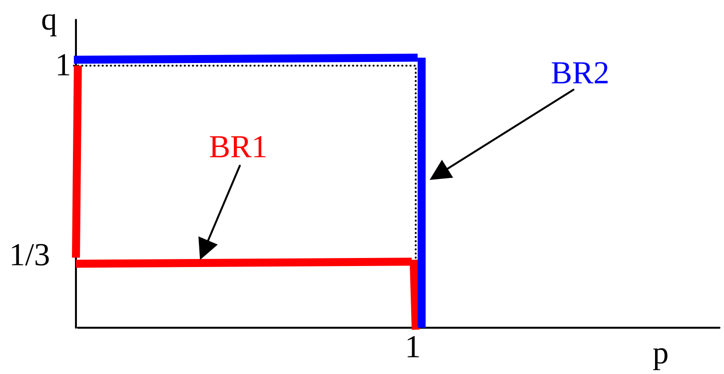
So:

$$\text{if } 1-3q > 0 \Leftrightarrow q < 1/3 \rightarrow p = 1$$
$$\text{if } 1-3q < 0 \Leftrightarrow q > 1/3 \rightarrow p = 0$$
$$\text{if } 1-3q = 0 \Leftrightarrow q = 1/3 \rightarrow 0 <= p <= 1$$

$$\text{Max }\{E[U_2] = 3(1-p)q\}$$
$$\text{Max }\{E[U_2] = 3q - 3pq\}$$
$$\text{Max }\{E[U_2] = q(3-3p)\}$$

So:

$$\text{if } 3-3p > 0 \Leftrightarrow p < 1 \rightarrow q = 1$$
$$\text{if } 3-3p < 0 \Leftrightarrow p > 1 \rightarrow \text{impossible}$$
$$\text{if } 3-3p = 0 \Leftrightarrow p = 1 \rightarrow 0 <= q <= 1$$



So ALL NE (in mixed strategy notation):

$$\{ (0, 0, 1), (1, 0); \quad (0, p, 1-p), (q, 1-q) \mid p = 1 \ \& \ q <= 1/3 \}$$

Students must include their zeros on actions given no weight. Note that one of the PSNE's is subsumed in the second MSNE.

d. (C,X) is a strict PSNE. (B,Y) is not.

2. (25%) Extensive Game.

a. Strategies for player 1: σ_1 = {A, B}. Strategies for player 2: σ_2= {MP, MQ, NP, NQ}

b. The game has 3 subgames. The one following 1's choice of A, the one following 1's choice of B, and the whole game.

c. When Y=1, working backwards, 2's BR are {MQ, NQ}. Then for each of these, $\sigma_1(\sigma_2=MQ)$ = {A,B}. And for $\sigma_1(\sigma_2=NQ)$ = {B}. So the set of SPNE are:

{ (A, MQ); (B,MQ); (B,NQ) }

d. When Y=0, working backwards, 2's BR are {MQ, NQ}. Then for each of these, $\sigma_1(\sigma_2=MQ)$ = {A}. And for $\sigma_1(\sigma_2=NQ)$ = {A,B}. So the set of SPNE are:

{ (A, MQ); (A,NQ); (B,NQ) }

3. (25%) Bertrand Competition.

a. **Since firm 1 has a lower marginal cost than firm 2, any NE will involve firm 1 taking the whole market while firm 2 will get nothing. There are two equilibria with discrete prices:**

$$\{P_1 = \$1.99, P_2 = \$2.00\}$$
$$\{P_1 = \$2.00, P_2 = \$2.01\}$$

In both equilibria, firm one wins the entire market and makes either \$0.99 or \$1.00 on the marginal unit. Also note that the demand specification is included in this problem solely because you needed to know that choke price (\$100) was greater than the highest marginal cost of any firm (\$2).

b. **With continuous prices, there is NO Nash Equilibrium. Firm 1 wants to undercut firm 2 by a very small amount but there is always a better deviation.**