

1. 40%

- a. False, strategies that are part of a NE are optimal *given* the strategy of the other player. Not *for all* strategies of the other player. A dominant strategy NE does have this property.
- b. $X \geq 1$. Note the “greater than or equal to” relationship. If player 1 plays T, player 2 must want to play R, ie $X \geq -2$ and $X \geq 1$.
- c. $Y < 4$. Note again the strict inequality condition.
- d. (T,L), (M,C). Solved using the underlining method.

2. 30%

- a. Player 1 moves at 3 nodes and player 2 moves at 2 nodes. They each have two possible actions at each so player 1 has 8 strategies and player 2 has 4. Thus we have strategies:
 $s_1 = \{PPP,PPS,PSP,SPP,SSP,SPS,PSS,SSS\}$
 $s_2 = \{PP,PS,SP,SS\}$
- b. SPNE: $\{(SSS), (SS)\}$. Solve via backward induction as always.
- c. False, only finite games (finite strategies and players) are guaranteed to have a SPNE. I gave credit for people that said, “True, if the game is finite...” However, strictly speaking, the statement is false because it does NOT hold for all extensive games.

3. 30%

- a. This is the stackelberg game that we solved in lecture with three firms. As with all extensive form games (with finite periods/moves), you have to solve backwards. So start with player 3, find his BR. Then move to player 2, plugging in player 3’s BR first (BEFORE taking your FOC!). Then go to player 1’s problem and again substitute in player 2 and player 3’s BR functions that you’ve already solved for. Player 1’s problem is only in terms of q_1 so you actually solve for a number. Thus, the Best Response Functions :

$$q_3(q_1, q_2) = \frac{1}{2}(16 - q_1 - q_2)$$

$$q_2(q_1) = \frac{1}{2}(16 - q_1)$$

$$q_1 = \frac{1}{2}(16) = 8$$

- b. All this asked was to take the BR functions in part (a) and solve them simultaneously. Given $q_1=8$, you could solve for q_2 . Given q_1 and q_2 , you could solve for q_3 . Thus:

$$q_1 = 8, q_2 = 4, q_3 = 2. \quad P = 16 - 8 - 4 - 2 = 2$$

- c. This is the “price leadership” or dynamic Bertrand game where firm’s sequentially choose their prices.

Technically, the SPNE is the limit as $\epsilon \rightarrow 0$, of the following:

$$\{ P_1 \text{ in } [0, \text{infinity});$$

$$P_2 = \min[P_1 - \epsilon, \text{Monopoly Price}];$$

$$P_3 = \min[\min [P_1, P_2] - \epsilon, \text{Monopoly Price}] }$$

However, since prices are continuous, ϵ is not defined and there is always a “better” deviation (like $P_2 = P_1 - 0.5 * \epsilon$). So like the 2 firm Bertrand game with equal and constant marginal cost, prices spiral down to cost, in this case zero. The only SPNE is thus:

$$(P_1 = P_2 = P_3 = 0)$$

As many of you mentioned, with finite players but infinite strategies, the Nash existence theorem does NOT guarantee an equilibrium to this game.

However, that does NOT mean that we don’t have one. Think of the 2 firm Cournot game: again firms choose quantities from a continuous set, but we still found a NE in that game as well.