Econ 414 Midterm – Solutions

1.

a. (T,B,L,R) all survive IESDS.
b. 2 Pure: (B,L) and (T,R) and 1 Mixed: [(0.5, 0.0, 0.5), (0.6, 0.0, 0.4)].

The mixed strategy is found by equalizing the expected conditional payoffs for each player. For player 1:

$-10q + 15(1-q) = 0 \rightarrow 15 = 25q \rightarrow q = 3/5$

For player 2 :

$-10p + 10(1-p) = 0 \rightarrow 10 = 20p \rightarrow p = ½$

c. See Osborne: The two conditions are that strategies in which players place a positive probability on must yield the same expected payoff while those that have zero probability placed upon them must NOT yield a higher expected value than those that do.

2.

a. Player I: [LLL, LLR, LRL, LRR, RRR, RRL, RLR, RLL] where the first letter is what player I plays in the period 1 while the second two letters describe what player I plays in period 3 in the left node and then in the right node.
   Player II: [LL, LR, RL, RR], again working from left to right.
b. By working backwards, we have 3 SPE: [(LLR, LR), (RLR, LR), (RLR, RR)]

3.

a. Discrete Prices. 2 Nash equilibria:
   (p_1 = \$2.00, [p_2 >= \$2.01, p_3 >= \$2.01, at least one with equality])
   (p_1 = \$1.99, [p_2 >= \$2.00, p_3 >= \$2.00, at least one with equality])

The reasoning for the second NE: firm 1 can always bid down the price below his opponents since he has the lowest marginal cost. There will never be an equilibrium in which firms 2 or 3 have any share of the market. Firm 1 need not lower his price to his cost since just undercutting firm 2's cost is enough to gain the whole market. Undercutting to \$1.99 while at least one or both of firms 2 and 3 set their prices equal to \$2 is one type of NE. Note player 1 cannot do any better: lowering his price lowers his profits, raising his price to \$2.00 or above will lower his profits because he must share the market with at least one other firm. Firms 2 and 3 also have no optimal deviation because they are earning zero profits and lowering their price will yield negative profits while raising their price will yield, again, zero profits.

The reasoning for the first NE: At this equilibrium, firms 2 and 3 are earning zero profits and firm 1 is making positive profits. For the same reason as above, firm 1 can do no better. Firm 2 could lower his price to \$2.00, yielding zero profits on half the market, and raising his price, again, yields zero profits.

b. Continuous Prices. There is NO NE. Firm 1 wants to set a price just below firm 2's cost and with continuous prices, there is no "smallest increment below $2.00," Firm 1 can always do a bit better but undercutting by half as much, say.

c. Yes, this is consistent because the Nash existence theorem only applies if both the number of players and number of strategies in the game are finite (in which case we call it a "finite game"). In (b), the strategy space for each player is infinite so the theorem is NOT violated and we need not have a NE.