

Economics 414 – Midterm SOLUTIONS

1. (35%) Consider the following simultaneous move game:

		Player 2	
		Q	R
Player 1	A	(0, 1)	(8, 2)
	B	(7, 1)	(6, 0)
	C	(8, 3)	(6, 2)

a. Define what is meant by a *strictly dominated strategy*.
See Osborne. A strategy is strictly dominated if it is inferior to another for EVERY strategy of the other player.

b. In the game above, determine if either player has a strictly dominated strategy. Be sure to state precisely *how* one or more strategies dominate another.

Player 1's action B is dominated by a mixture of A and C as follows: Suppose player 1 puts probability p , & 1- p on A & C. If player 2 plays Q, we require $0p+8(1-p) > 7$. Or $8p < 1$. Or $p < 1/8$. If player 2 plays R, we require $8p + 6(1-p) > 6$. This holds for all $p > 0$. Thus, for $0 < p < 1/8$, the mixture of A and C dominates B. Player 2 does not have a strictly dominated strategy.

c. Find *all* Nash Equilibria of the game above.

Given that players never play strictly dominated strategies, we can remove action B from consideration. Now, by the underlining method, we see that there are two pure strategy NE at (C,Q) and (A,R). There is also a mixed strategy NE which can be found as follows:

$$E[U1] = 0pq + 8p(1-q) + 8(1-p)q + 6(1-p)(1-q)$$

$$= p(2-10q) + 2q + 6$$

$$\rightarrow \text{breakpoint at } q = 1/5$$

$$E[U2] = 1pq + 2p(1-q) + 3(1-p)q + 2(1-p)(1-q)$$

$$= q(1-2p) + 2$$

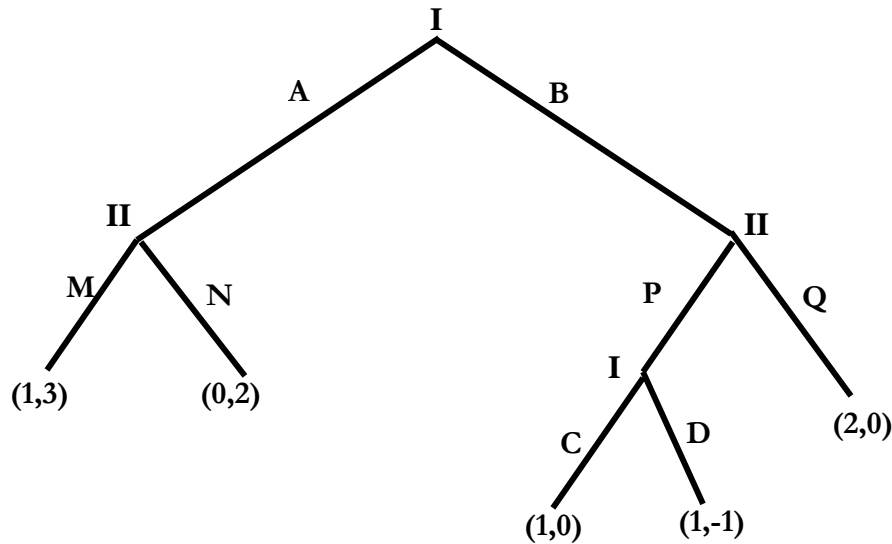
$$\rightarrow \text{breakpoint at } p = 1/2$$

So there is a MSNE is at $(1/2, 0, 1/2), (1/5, 4/5)$.

All NE (in mixed strategy notation) are therefore:

$$NE = \{ ((0,0,1), (1,0)); ((1,0,0), (0,1)); ((0.5, 0, 0.5), (0.2, 0.8)) \}$$

2. (35%) Consider the following extensive game:



Payoffs are denoted (X,Y) where X is player I's payoff and Y is player II's payoff.

a. Write down all possible strategies of each player.

$$\sigma_1 = (AC, AD, BC, BD)$$

$$\sigma_2 = (MP, MQ, NP, NQ)$$

b. Solve for the Subgame Perfect Nash Equilibrium of the extensive game.

Note that player 1 is indifferent at the bottom node. Player 2 always plays M at the left node and is sometimes indifferent at the right node. So consider the two potential Nash strategies of player 2, MP and MQ.

$$\text{If } \sigma_2 = MP \rightarrow \sigma_1 = (AC, AD, BC, BD)$$

$$\text{If } \sigma_1 = AC \rightarrow \sigma_2 = (MP, MQ)$$

$$\text{If } \sigma_1 = AD \rightarrow \sigma_2 = (MQ)$$

$$\text{If } \sigma_1 = BC \rightarrow \sigma_2 = (MP, MQ)$$

$$\text{If } \sigma_1 = BD \rightarrow \sigma_2 = (MQ)$$

We have SPNE's if we end up where we started so there are SPNEs at: $((AC, MP); (BC, MP))$.

$$\text{If } \sigma_2 = MQ \rightarrow \sigma_1 = (BC, BD)$$

$$\text{If } \sigma_1 = BC \rightarrow \sigma_2 = (MP, MQ)$$

$$\text{If } \sigma_1 = BD \rightarrow \sigma_2 = (MQ)$$

We have SPNE's if we end up where we started so there are SPNEs at: $((BC, MQ); (BD, MQ))$.

So ALL NE are $\{ (AC, MP); (BC, MP); (BC, MQ); (BD, MQ) \}$

- c. In general, explain the difference between Nash Equilibrium and Subgame Perfect Nash Equilibrium. Why is the former insufficient for dynamic games?

See Osborne. The NE concept is applied at the start of the game and does not take into account what players do at each point in the (dynamic) game. NE allows for non-credible threats as in the entry game. SPNE requires that players act optimally in EVERY subgame versus NE which only require players to act optimally at the beginning of the game (ignoring the dynamics).

3. (30%) Consider 2 oligopolists facing the (inverse) demand curve $P = 12 - Q$, where $Q = q_1 + q_2$. Assume that $C_i(q_i) = q_i^2$ for $i = 1, 2$.

- a. Assume that firms compete via Stackelberg in quantities with firm 1 moving first. Find the Subgame Perfect Nash Equilibrium.

With firm 1 moving first, we work backwards and solve firm 2's problem first:

$$\begin{aligned} & \text{Max}(q_2) \{q_2 * P(Q) - q_2^2\} \\ & \text{Max}(q_2) \{q_2(12 - q_1 - q_2) - q_2^2\} \\ & \text{FOC}(q_2) : 0 = 12 - q_1 - 2q_2 - 2q_2 \\ & q_2(q_1) = 1/4(12 - q_1) \end{aligned}$$

Next consider firm 1's problem :

$$\begin{aligned} & \text{Max}(q_1) \{q_1 * P(Q) - q_1^2\} \\ & \text{Max}(q_1) \{q_1 * (12 - q_1 - 1/4 * (12 - q_1)) - q_1^2\} \\ & \text{FOC}(q_1) : 0 = 12 - 2q_1 - 3 + 1/2q_1 - 2q_1 \\ & 3.5q_1 = 9 \\ & q_1 = 9/3.5 = 18/7 = 2.571 \text{ (approx)} \end{aligned}$$

Plug this into firm 2's BR :

$$q_2(q_1) = 1/4(12 - (18/7)) = 33/14 = 2.357 \text{ (approx)}$$

Thus, the SPNE is : (18/7, 33/14)

- b. Suppose instead that firms are allowed to collude and make their decisions cooperatively at the same time. Find the profits of each firm if they collude.

If collusion is possible, firms solve the following problem:

$$\begin{aligned} & \text{Max}(q_1, q_2) \{(q_1 + q_2)(12 - q_1 - q_2) - q_1^2 - q_2^2\} \\ & \rightarrow \text{FOC}(q_1) : 0 = 12 - 2q_1 - q_2 - q_2 - 2q_1 \rightarrow q_1 = 3 - 1/2 * q_2 \\ & \rightarrow \text{By symmetry, FOC}(q_2) \rightarrow q_2 = 3 - 1/2 * q_1 \\ & \rightarrow q_1 = 3 - 1/2 * (3 - 1/2 * q_1) \rightarrow q_1 = 2 \\ & \rightarrow q_2 = 2 \\ & \rightarrow P = 12 - 2 - 2 = 8 \\ & \rightarrow \pi_i = 2 * 8 - 2^2 = 16 - 4 = 12 \end{aligned}$$

Note that $a^2 + b^2 \neq (a + b)^2$ which is why the costs are included as above.