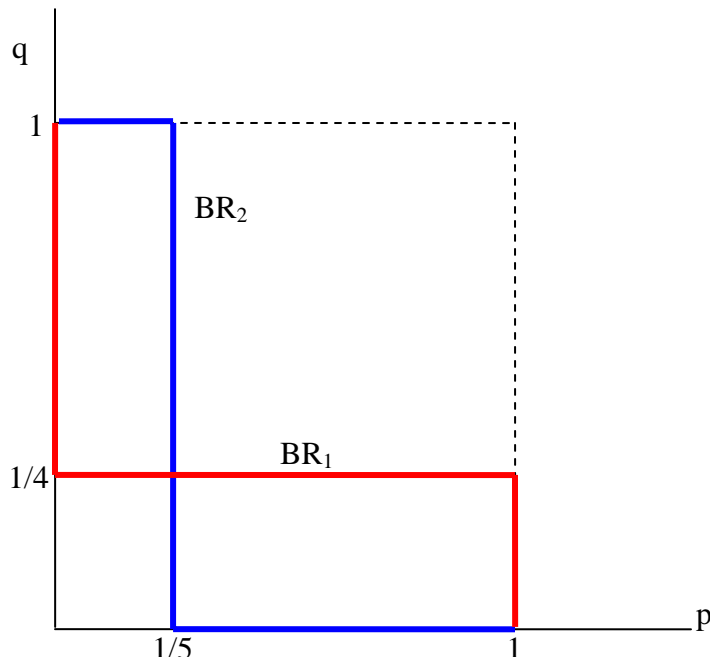


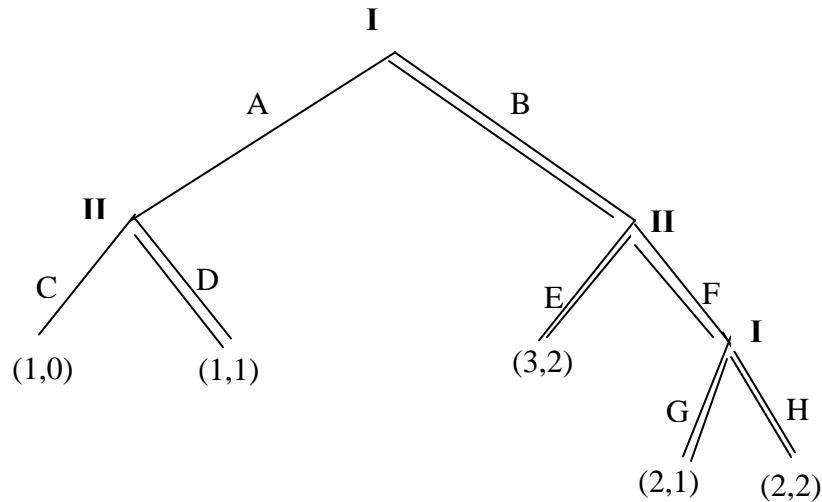
1. 40% of total

- a. See lecture notes.
- b. If player 1 plays T, player 2 gets $0 \cdot q + 4(1-q)$ from playing (q,1-q,0) on his three strategies. Playing R yields 2. We require $4(1-q) > 2$, or $q < \frac{1}{2}$. If player 1 plays M, player 2 gets $2q + 1(1-q)$ from mixing and 1 from playing R alone. $q > 0$ is required. Finally, if player 1 plays B, player 2 gets 4 from playing any mixture of L and C while only 3 from playing R. Thus mixing on (L,C) with probabilities (q,1-q) dominates R if $0 < q < \frac{1}{2}$.
- c. First note that B is dominated by M for player 1. We are left with just two strategies for each player (T and M for player 1 and L and C for player 2). The underlining payoffs method yields two pure strategy NE at (T,C) and (M,L). Equalizing conditional expected payoffs, or equivalently maximizing unconditional payoffs yields a MSNE of $(\frac{1}{5}, \frac{4}{5}, 0), (\frac{1}{4}, \frac{3}{4}, 0)$. Thus all NE are

$$[(1, 0, 0), (0, 1, 0)] ; [(0, 1, 0), (1, 0, 0)] ; [(\frac{1}{5}, \frac{4}{5}, 0), (\frac{1}{4}, \frac{3}{4}, 0)] \}$$



2. 30% of total
 a. See lecture notes.



- b. Working from top to bottom and left to right, $\sigma_1 = \{AG, AH, BG, BH\}$ and $\sigma_2 = \{CE, CF, DE, DF\}$.
- c. Working backwards, we see that player I is indifferent at his last node between G and H, while he only plays B at the first node in the game. His optimal strategies are (BG) and (BH). Player II always plays D at the left node while may play E or F depending on player I's strategy. In particular, if player I plays (BG), player II will play (DE) and if player I plays (BH), then player II will want to play (DE) or (DF). We can summarize this as follows:
- i. $\sigma_1 = (BG) \rightarrow \sigma_2 = (DE)$
 - ii. $\sigma_1 = (BH) \rightarrow \sigma_2 = (DE), (DF)$
 - iii. $\sigma_2 = (DE) \rightarrow \sigma_1 = (BG), (BH)$
 - iv. $\sigma_2 = (DF) \rightarrow \sigma_1 = (BG), (BH)$

So we have the following mutual best responses (I.e., Sub-game Perfect Nash Equilibria):

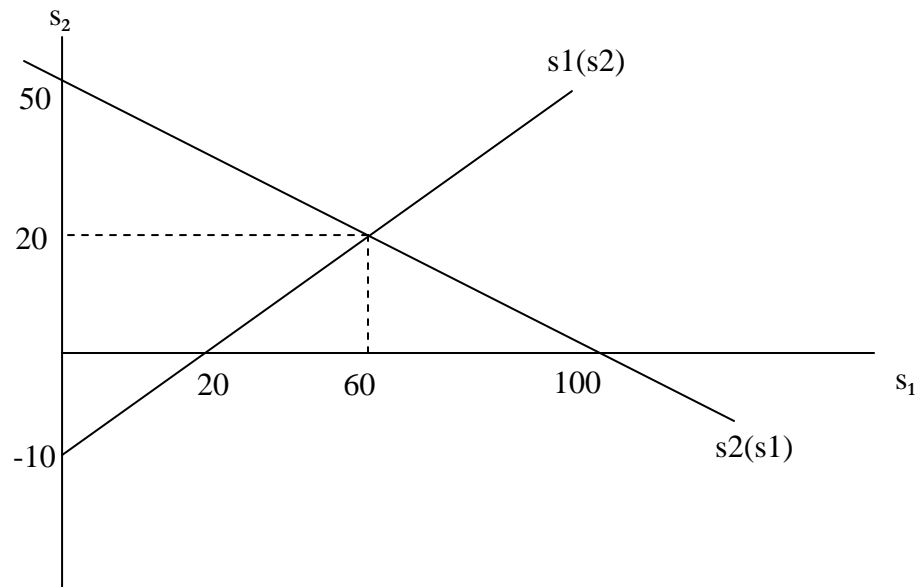
$$\{(BG, DE); (BH, DE); (BH, DF)\}$$

Note that even though (BG) is a best response for player 1 to (DF) for player 2, DF is NOT a best response to BG.

3. This is just the Cournot game in part (a) and the Stackelberg game in part (b).

- a. FOC(s_1): $40 - 2s_1 + 4s_2 = 0 \rightarrow s_1(s_2) = 20 + 2s_2$
 FOC(s_2): $100 - 2s_2 - s_1 = 0 \rightarrow s_2(s_1) = 50 - 0.5s_1$
 Note that both SOC < 0 so we will be at a maximum.

Graph:



The two FOCs above are just two equations/ two unknowns and solving them simultaneously (plugging one into the other), yields the equilibrium, $(s_1, s_2) = (60, 20)$, as shown in the graph.

- b. Since player 1 moves first and 2 moves second (after observing player 1's choice), we already know that player 2 will choose a level of s_2 off his best response curve: $s_2(s_1) = 50 - 0.5s_1$. So in period 1, player 1 maximizes his payoff but takes player 2's best response curve into account (just as in Stackelberg). So player 1 maximizes:

$$\pi_1(s_1, s_2(s_1)) = 40s_1 - s_1^2 + 4s_1 \cdot [50 - 0.5s_1]$$

$$\text{FOC}(s_1): 40 - 2s_1 + 200 - 4s_1 = 0 \rightarrow s_1 = 40$$

Plug $s_1 = 40$ into player 2's BR curve: $s_2(40) = 30$. Thus the equilibrium outcome is $(s_1, s_2) = (40, 30)$.

- c. The payoffs for players 1 and 2 respectively in part (a) are (3600, -2600) and in part (b) they are (4800, -1100). The argument is similar to a "revealed preference" argument. Player 1 can always just choose the quantity we solved for in part (a), $s_1 = 60$ and attain the payoff 3600. Since he's now optimizing his payoff in this dynamic game where he knows that player 2 will get to react to his choice, he has to do at least as well in part (b) [since the part (b) strategy is still an option]. If

doing something other than $s_1=60$ else yielded less than 3600, then clearly $s_1=60$ would be a better strategy. So player 1 has a first mover advantage. It's interesting that player 2 also does better in the dynamic game. Note in dynamic homogeneous good Bertrand, the first mover does not have this advantage. However, he can do no worse than he does in the simultaneous Bertrand setting.