

## 1 Mathematics Review - Questions

- (1) Find the first and second derivative of the following expression:

$$f(x) = 3x^2 + 2x + 6$$

- (2) Find the first derivative of the following expression:

$$h(x) = (3 - x)(2 + x)^2$$

- (3) Find the partial derivative with respect to  $x$ ,  $\frac{\partial g(x, y)}{\partial x}$ , and the partial derivative with respect to  $y$ ,  $\frac{\partial g(x, y)}{\partial y}$  of the following expression:

$$g(x, y) = \alpha x^3 + \beta yx$$

- (4) Solve for  $x^*$  and  $y^*$  that satisfy the following two equations:

$$2x + 3y = 8$$

$$y - x = 1$$

- (5) Consider the following expression:

$$f(x) = (2 - x)^2 + 3$$

Find all optimum of the expression (where the first derivative or the “first order condition” [FOC] is equal to zero). Is the optimum a maximum or a minimum?

- (6) Suppose a random variable,  $X$ , is distributed uniformly on the interval from 0 to 2, i.e.  $X \sim U[0, 2]$ . What is the expected value of  $X$ ,  $E[X]$ ? Find  $E[X|X \leq 1.5]$ , the expected value of  $X$  given that it is not more than 1.5.

## 2 Mathematics Review - Solutions

- (1)  $f'(x) = 6x + 2$ .  $f''(x) = 6$ .
- (2)  $h'(x) = -(2 + x)^2 + 2(3 - x)(2 + x) = (2 + x)(4 - 3x)$ .
- (3)  $\frac{\partial g}{\partial x} = 3\alpha x^2 + \beta y$ .  $\frac{\partial g}{\partial y} = \beta x$ .
- (4)  $x^* = 1$ ,  $y^* = 2$ .
- (5)  $FOC(x) : -2(2 - x) = 0$ ,  $x^* = 2$ .  $SOC(x) = 2 > 0$ ,  
Minimum.
- (6)  $E[X] = 1$ ,  $E[X|X \leq 1.5] = 0.75$ .