Macroeconomics I
Michaelmas Term

Matthew Chesnes
The London School of Economics

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1 Week 1: 10/8 - 10/12

- The aggregate demand curve represents the simultaneous equilibrium of money (LM) and commodity markets (IS) $\longrightarrow (Y, R)$ combinations.
- Market for goods and services (IS):

$$y \equiv \text{Real GDP} = \text{Real Income} \equiv \text{Supply of output.}$$

$$y = c + i + g.$$

- Consumption:

$$c = c(\text{Disposable income} = y^d, \text{Wealth or assets} = a).$$

Both y^d and a depend positively on c.

$$c = c[y - \tau_0 - \tau_1 y, \frac{M+B}{p} + kq].$$

Where y is total income, τ_0 is lump sum taxes, τ_1 is a proportional tax, M and B are nomimal money and bonds, k is the real stock of capital, and q is the real stock price.

- Investment:

$$i = i(\text{Real interest rate} = r, y, \text{Real capital stock} = k).$$

$$i = i(R - \hat{p}^e, y, k).$$

- Bringing it all together, we get the total IS curve:

$$y = c[y - \tau_0 - \tau_1 y, \frac{M+B}{p} + kq] + i[R - \hat{p}^e, y, k] + g.$$

Taking the total differential,

$$dy = c_y[(1-\tau_1)dy - d\tau_0 - y d\tau_1] + c_a[\frac{dM + dB}{p} - (m+b)\frac{dp}{p} + qdk + kdq] + i_r[dR - d\hat{p}^e] + i_y dy + i_k dk + dg.$$

2 Week 2: 10/15 - 10/19

- Solving the IS equation for dR and dy, one obtains,

$$\frac{dR}{dy} = \frac{1 - c_y(1-\tau_1) - i_y}{i_r} = \frac{\alpha}{i_r} < 0.$$

Also,

$$\frac{dy}{dg} = \frac{1}{1 - c_y(1-\tau_1) - i_y} = \frac{1}{\alpha} > 0.$$

2.1 Asset Market

- Money Supply:

$$\frac{M^s}{p} = m^d[R_m(+), R_b(-), R_k(-), y(+), a(+)].$$

With effects on money demand (+/-).

- Bond Supply:

$$\frac{B^s}{p} = b^d[R_m(-), R_b(+), R_k(-), y(+), a(+)].$$

- Equity Supply:

$$qk^s = k^d[R_m(-), R_b(-), R_k(+), y(+), a(+)].$$

- Assume bonds and equities are perfect substitutes and therefore $R_b = R_k = R$. If money provide no return, $R_m = 0$. Thus,

$$\frac{M^s}{p} = m[R, y, a].$$

- Taking the total differential,

$$\frac{dM}{p} - m\frac{dp}{p} = m_R dR + m_y dy + m_a[\frac{dM + dB}{p} - (m+b)\frac{dp}{p} + qdk + kdq].$$

- Once again by solving for dR and dy, one obtains,

$$\frac{dR}{dy} = \frac{-m_y}{m_R} > 0.$$

Also,

$$\frac{dy}{dM} = \frac{1 - m_a}{pm_y} > 0.$$

2.2 IS/LM Analysis

- The two total differentials of IS and LM can be simplified by putting them into matrix form.

$$\begin{bmatrix} \alpha & -i_r \\ m_y & m_R \end{bmatrix} \begin{bmatrix} dy \\ dR \end{bmatrix} = \tag{1}$$

$$\begin{bmatrix} 1 & -c_y & -yc_y & \frac{c_a}{p} & \frac{c_a}{p} & -i_r & \frac{-c_a(m+b)}{p} \\ 0 & 0 & 0 & \frac{1-m_a}{p} & -\frac{m_a}{p} & 0 & -\frac{m-m_a(m+b)}{p} \end{bmatrix} \begin{bmatrix} dg \\ d\tau_0 \\ d\tau_1 \\ dM \\ dB \\ d\hat{p}^e \\ dp \end{bmatrix}. \tag{2}$$

- Thus, via cramers rule,

$$\frac{dy}{dg} = \frac{\begin{vmatrix} 1 & -i_r \\ 0 & m_R \end{vmatrix}}{\begin{vmatrix} \alpha & -i_r \\ m_y & m_R \end{vmatrix}} = \frac{m_R}{\alpha m_R + m_y i_r} = \frac{1}{\alpha + \frac{m_y}{m_R} i_r}. \tag{3}$$

- Since $\frac{1}{\alpha}$ is the standard Keynesian multiplier, here it is decreased by $\frac{m_y}{m_R} i_r$ due to the financial crowding out effect. Let $\beta = \frac{m_y}{m_R} i_r$ and the multiplier becomes,

$$\frac{dy}{dg} = \frac{1}{\alpha + \beta} > 0.$$

[G-2.1]

- Another multiplier, via cramers rule,

$$\frac{dy}{dM} = \frac{\begin{vmatrix} \frac{c_a}{p} & -i_r \\ 1 - \frac{m_a}{p} & m_R \end{vmatrix}}{\begin{vmatrix} \alpha & -i_r \\ m_y & m_R \end{vmatrix}} = \frac{c_a m_R + i_R(1 - m_a)}{p\Delta} = \frac{c_a + \frac{i_r}{m_R}(1 - m_a)}{p(\alpha + \beta)} <> 0. \tag{4}$$

- Another multiplier:

$$\frac{dR}{dg} = \frac{-m_y}{m_R(\alpha + \beta)} > 0.$$

- Consider adding a balanced budget constraint: $dg = \tau_1 dy + d\tau_0$. Rewrite the system as,

$$\begin{bmatrix} \alpha & -i_r & c_y \\ m_y & m_R & 0 \\ \tau_1 & 0 & 1 \end{bmatrix} \begin{bmatrix} dy \\ dR \\ d\tau_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} dg. \tag{5}$$

Thus,

$$\frac{dy}{dg} = \frac{1 - c_y}{\alpha' + \beta}.$$

Where $\alpha' = 1 - c_y - i_y$, reflects the impact of taxes on consumption.

- Now consider the case where monetary policy takes the form of open market operations. Thus $dM + dB = 0$. If this is the case, there are no wealth effects, $c_a = m_a = 0$. Thus, eliminating those terms from the equation above yields,

$$\frac{dy}{dM} = \frac{i_r}{m_R} \frac{1}{P(\alpha + \beta)}.$$

- We can also compare the effectiveness of monetary and fical policy together:

$$\frac{\frac{dy}{dM}}{\frac{dy}{dg}} = \frac{\frac{i_r}{m_R}\frac{1}{P(\alpha+\beta)}}{\dfrac{1}{\alpha + \beta}} = \frac{i_r}{pm_R}.$$

  Thus, the only effective shocks are those that change the sensitivity of money or investment demand to the interest rate.

- What happens when there is a rise in the expected inflation in an economy. Solving the system above by cramers rule,

$$\frac{dy}{d\hat{p}^e} = \frac{-i_r}{\alpha + \beta} > 0.$$

  This happens because IS shifts out in response to a the higher expected prices **[G-2.2]** :

$$\frac{dR}{d\hat{p}^e} = \frac{\beta}{\alpha + \beta} > 0.$$

  Thus the nominal interest rate must rise less than the rise in expected inflation, yielding a lower real interest rate and more investment.

- Now, consider the slope of the aggregate demand curve which can again be derived from the IS/LM system.

$$\frac{dy}{dp} = \frac{-c_a(m + b) - [m - m_a(m + b)]\frac{i_r}{m_R}}{p(\alpha + \beta)}.$$

  The first term in the numerator is the "Tobin Effect" and the second is the "Keynes effect."

3 Week 3: 10/22 - 10/26

3.1 Aggregate Supply

- Production function: $y_i = F(l_i, k_i)$. Assume $F_L > 0$, $F_k > 0$, $F_{LL} < 0$, $F_{kk} < 0$, $F_{lK} > 0$. Assume also that the firms are competitive on the output and input side and capital stock is fixed in the short run. Thus labour is the only decision variable.

- Firm's Problem: Max $\pi = pF(L_i, \bar{k}_i) - WL_i$. Thus FOC:

$$\frac{W}{p} = F_L(L_i, k_i).$$

  Or the real wage equals the marginal product of the labor hired.

- Thus labor demand $= W = pF_L(L_i, k_i)$. Since all firms are identical, they all produce an equal share of the market. Thus,

$$\frac{W}{p} = F_L(\frac{L}{n}, \frac{k}{n}).$$

  For each of the n firms.

- Since we have constant returns to scale, $F(L_i, k_i)$ is homogeneous of degree 1 and therefore $F_L(L_i, k_i)$ is homogeneous of degree 0. Thus,

$$\frac{W}{p} = F_L(\frac{L}{n}, \frac{k}{n}) = F_L(L, k).$$

  Thus a single firm can represent the entire economy.

- Labor Supply:

$$L^s = l(\frac{W}{p}).$$

  And we'll assume the substitution effect dominates so that an increase in the real wage will lead to an increase in the labor supply.

- In the classical case, a shock to labor demand is offset by an equal shock to labor supply which means that employment stays fixed and wage and prices move freely. [**G-3.1**] Thus the AS curve is vertical.

- Equilibrium conditions in the labor market:

$$\frac{W}{p} = F_L(L, k).$$

$$L = l(\frac{W}{p}).$$

  Substituting,(*)

$$L = l[F_L(L, \bar{k})].$$

- Implicitly, we can define labor as a function of capital such that $L = g(k)$ where $g'(k) > 0$.
- If we totally differentiate $(*)$,

$$dL = l' F_{LL} dL + l' F_{Lk} dk.$$

Thus,

$$\frac{dL}{dk} = \frac{l' F_{Lk}}{1 - l' F_{LL}} = g' > 0.$$

- Thus $y = F(L, k) = F(g(k), k) = y^c(k) \equiv$ The classical supply function. Also,

$$\frac{dy}{dk} = F_L g' + F_k > 0.$$

$g'(k)$ represents how much labor responds to capital which in return tells us, through $y^c(k)$, how much output responds to capital.

- Money is said to be superneutral if a change in the rate of monetary growth has no real effects, ie if $c_a = 0$ or $m_R = 0$.

3.2 Keynesian Supply

- In the Keynesian case, we have a fixed labor supply depending on nominal wage. Wages and prices are sticky. Thus,

$$\frac{\bar{W}}{p} = F_L(L, k).$$

Therefore,

$$L = h(\frac{\bar{W}}{p}, k).$$

Totally differentiating L,

$$dL = h_1 dF_L + h_2 dk.$$

Expanding dF_L,

$$dL = h_1 [F_{LL} dL + F_{Lk} dk] + h_2 dk.$$

Now to solve for h_1, or $\frac{\partial L}{\partial \frac{W}{p}}$, set $dk = 0$,

$$dL = h_1 [F_{LL} dL + \underbrace{F_{Lk} dk}_{0}] + \underbrace{h_2 dk}_{0}.$$

$$dL = h_1 F_{LL} dL.$$

Divide by dL,

$$1 = h_1 F_{LL}.$$

Thus,

$$h_1 = \frac{1}{F_{LL}} < 0.$$

To find h_2, solve the equation above for dL. Thus,

$$dL = \frac{h_1 F_{Lk} dk + h_2 dk}{1 - h_1 F_{LL}}.$$

Divide by dk,

$$\frac{dL}{dk} = \frac{h_1 F_{Lk} + h_2}{1 - h_1 F_{LL}}.$$

Note that $h_2 = \frac{dL}{dk}$. Thus,

$$h_2 = \frac{h_1 F_{Lk} + h_2}{1 - h_1 F_{LL}}.$$

Solve for h_2,

$$h_2 - h_1 h_2 F_{LL} = h_1 F_{Lk} + h_2.$$
$$h_2(1 - h_1 F_{LL} - 1) = h_1 F_{Lk}.$$
$$h_2 = -\frac{h_1 F_{Lk}}{h_1 F_{LL}}.$$

Thus finally,

$$h_2 = -\frac{F_{Lk}}{F_{LL}} > 0.$$

And thus in the Keynesian case,

$$y^k(\frac{\bar{W}}{p}, k) = F[\underbrace{h(\frac{W}{p}, k)}_{L}, k].$$

Where,

$$y_1^k = \frac{\partial y^k}{\partial \frac{\bar{W}}{p}} = F_L h_1 < 0.$$

$$y_2^k = \frac{\partial y^k}{\partial k} = F_L h_2 + F_k > 0.$$

- In the classical case, output does not change with fiscal policy, though the composition might. Variables of interest are therefore p and R. Fiscal policy shocks ($dg > 0$), results in $dR > 0$, $dp > 0$, and $dy = 0$. Monetary policy shocks ($dM > 0$), results in $dR < 0$, $dp > 0$, and $dy = 0$.

- Equilibrium in the Keynesian case gives us:

$$\frac{dy}{dg} = \frac{1}{\alpha + \beta + \gamma}.$$

Where $\beta \equiv$ financial crowding out and $\gamma \equiv$ real crowding out.

3.3 Imperfect Competition and the Weitzman Model

- Prices NOT taken as given. $MR = MC$ is optimal condition here.
- Firm output, y_i, a function of the relative prices, the elastisticity of demand, ϵ, and average market output.

$$y_i = (\frac{P_i}{P})^{-\epsilon}\frac{y}{n}.$$

- Labor is the only factor of production and there are constant returns to scale. Thus,

$$y_i = L_i.$$

- The firm's objective it to maximize:

$$(Pi - W)(\frac{P_i}{P})^{-\epsilon}\frac{y}{n}.$$

  Which yields FOC:

$$Pi = W\frac{\epsilon}{\epsilon - 1}.$$

  The price is thus a constant mark up over the wage and is independent of demand and competitors prices.

- The aggregate price level is $P = W\dfrac{\epsilon}{\epsilon - 1}$.
- Thus from above, $y_i = \frac{y}{n}$ and $y = L$.
- Under fixed wages (Keynesian case), there is a fixed wage, $\bar{W}$, and the AS curve is horizontal. Under flexible wages (Classical case),

$$L = l[\frac{W}{P} = \frac{\epsilon - 1}{\epsilon}] = y.$$

  So that the AS curve is vertical (but at a lower level of activity ?)

4 Week 4: 10/29 - 11/01

4.1 The Open Economy

- **[G-4.1]** Review: In the Keynesian world, the AS curve slopes upward and shifts because of changes in the capital stock or in the wage rate. In the Classical world, AS is vertical and shifts only because of changes to the capital stock.

- In the open economy, we need to introduce a few new variables:

$$\begin{aligned}
c^d &\equiv \text{Consumption of domestic goods (net of imports)} \\
x &\equiv \text{Exports} \\
e &\equiv \text{Price of foreign currency } (e \uparrow \Rightarrow \text{depreciation}) \\
P^* &\equiv \text{World Price level in foreign currency} \\
s &\equiv \text{Real exchange rate / measure of competitiveness } (s = e\tfrac{P^*}{P}) \\
y_d &\equiv \text{Disposable income in units of domestic goods } (y^d = y(1-\tau_1) - \tau_0) \\
a &\equiv \text{Real wealth in units of domestic goods } (a = \tfrac{M+B}{P} + qk)
\end{aligned}$$

- Thus on the demand side, IS:

$$y = c^d(s(?), y_d(+), a(+)) + i(R - \hat{P}^e(-), y(+), k) + g + x(s(+), y^*(+)).$$

- In general, we assume that $\dfrac{\partial c}{\partial s} > 0$ because the substition effect dominates the income effect. Thus as the economy becomes more competitive, foreigners demand more of our goods and we demand less of foreign goods.

- LM:
$$\frac{M}{P} = m[R, y, a].$$

- Slope of IS:
$$\frac{dR}{dy} = \frac{\alpha^*}{i_r}.$$
Where $\alpha^* = 1 - c^d_y(1-\tau_1) - i_y$. Since $c^d_y < c_y$, $\alpha^* > \alpha$ and therefore IS is steeper in the open economy then in the closed one.

- Also, another multiplier:
$$\frac{dy}{dg} = \frac{1}{\alpha^*} < \frac{1}{\alpha}.$$
So fiscal policy is less effective in the open economy.

- Note that $s = e\dfrac{P^*}{P}$.

- Differentiating the new IS curve,

$$
\begin{aligned}
dy &= c_s^d[\frac{e}{p}dP^* + \frac{P^*}{P}de - e\frac{P^*}{P^2}dP] \\
&+ c_{y_d}^d[(1-\tau_1)dy - d\tau_0 - yd\tau_1] \\
&+ c_a^d[\frac{dM+dB}{p} - (m+b)\frac{dp}{p} + qdk + kdq] \\
&+ i_r[dR - d\hat{p}^e] \\
&+ i_y dy \\
&+ i_k dk \\
&+ dg \\
&+ x_s[\frac{e}{p}dP^* + \frac{P^*}{P}de - e\frac{P^*}{P^2}dP] \\
&+ x_{y^*}dy^*.
\end{aligned}
$$

- And as a reminder, the total differential of the LM curve:

$$\frac{dM}{p} - m\frac{dp}{p} = m_R dR + m_y dy + m_a[\frac{dM+dB}{p} - (m+b)\frac{dp}{p} + qdk + kdq].$$

- Or in matrix form,

$$\begin{bmatrix} \alpha^* & -i_r \\ m_y & m_R \end{bmatrix} \begin{bmatrix} dy \\ dR \end{bmatrix} = \begin{bmatrix} c_s^d + x_s & \frac{-c_a^d(m+b)}{P} \\ 0 & \frac{m_a(m+b)-m}{P} \end{bmatrix} \begin{bmatrix} ds \\ dP \end{bmatrix}. \tag{6}$$

- Thus solving for the slope of the AD curve, (via cramer)

$$\frac{\partial P}{\partial y} = \frac{-P(\alpha^* + \beta)}{\frac{i_r[m - m_a(m+b)]}{m_R} + c_a^d(m+b) + (c_s^d + x_s)s}.$$

Or,

$$\frac{\partial P}{\partial y} = \frac{-P(\alpha^* + \beta)}{\text{Keynes Effect + Wealth Effect + Competitiveness Effect}}.$$

- Both the numerator and denominator in this last equation are greater than in the closed economy, so the effect on the slope of AD is ambiguous.

4.2 Aggregate Supply in the Open Economy

- Production function : $y = F(L;k)$, profit maximization yields, $\frac{W}{P} = F_L(L^D;k) \equiv$ labor demand.

- Labor supply in the classical case is a function of the nominal wage deflated by foreign and domestic prices:

$$L^s = l\Big[\frac{W}{P^{1-\theta}(eP^*)^\theta}\Big].$$

Where θ is the import share of the economy.

- Since $s = e(\frac{P^*}{P})$,

$$L^s = l\Big[\frac{W}{P} * s^{-\theta}\Big].$$

- $L^s = l[\frac{W}{P}(+), s(-)]$. So a higher real exchange rate (less competitive economy) leads to a lower supply of labor.

- Substituting the demand condition in the supply,

$$L = l[F_L * s^{-\theta}].$$

- Totally differentiating,

$$dL = l' s^{-\theta} F_{LL} dL - \theta s^{-(\theta+1)} l' F_L ds.$$

$$dL = \frac{-\theta s^{-(\theta+1)} l' F_L ds}{1 - l' s^{-\theta} F_{LL}}.$$

$$\frac{dL}{ds} = \frac{-\theta s^{-(\theta+1)} l' F_L}{1 - l' s^{-\theta} F_{LL}} < 0.$$

- Thus $L = f[k, e(-), P^*(-), P(+)]$ and $y = g[k(+), e(-), P^*(-), P(+)]$. Thus in the open economy classical case, the AS curve is upward sloping because the trade improvement reduces product wages more than consumption wages. The effect will be small if labor supply is inelastic.

5 Week 5

- More on the open economy. Consider the IS and LM definitions which together describes aggregate demand:

$$IS : y = c^d(y, a, \underbrace{s}_{+}) + i(r, y, k) + g + x(\underbrace{s}_{+}, \underbrace{y^*}_{+}).$$

  Where y^* is foreign output.

$$LM : \frac{M}{P} = m^d(R^*, y, a).$$

- And aggregate supply is:

$$AS = \begin{cases} y_k = y(P, W, k) \\ y_c = y(s) = y(P, P^*, e) \end{cases} \tag{7}$$

- And we know,

$$\frac{dy_k}{dP} = y_k' > 0.$$

$$\frac{dy_c}{dP} = \frac{dy}{ds}\frac{ds}{dP} = y_c'(-\frac{s}{P}).$$

  Because $L = l(\frac{W}{P^{1-\theta}eP^{*\theta}}) = l(\frac{W}{P}s^{-\theta})$ and $s = \dfrac{eP^*}{P}$, therefore $\frac{ds}{dP} = \frac{-eP^*}{P^2} = -\frac{s}{P}$.

- So now we want to go on and consider 4 cases of the international economy in terms of exchange rate policy and the supply assumptions. See notes for all the graphs showing the shifts in each of these models. **[G-5.1]** For notation, denote,

$$\varphi = \frac{dy}{dg},$$

$$\mu = \frac{dy}{dM}.$$

  - Case I: Fixed exchange rate § Keynesian supply. An increase in government spending shifts out the IS curve but since the exchange rate must remain fixed, the central bank comes in and increases the money supply to keep the interest rate fixed. Thus LM shifts out. Overall,

$$\varphi > 0.$$

$$\mu = 0.$$

    Therefore the LM curve is endogenous so only consider IS and AS when solving. (y, P, M) are the endogenous variables. When the world interest rate, R^*, falls, this is expansionary at home.

- Case II: Fixed exchange rate § Classical supply. An increase in governement spending shifts out the IS curve but since the exchange rate must remain fixed, the central bank comes in and increases the money supply to keep the interest rate fixed. Thus LM shifts out. Overall, the effects are exactly the same as in case I,

  $$\varphi > 0.$$

  $$\mu = 0.$$

  Therefore the LM curve is endogenous so only consider IS and AS when solving. (y, P, M) are the endogenous variables. When the world interest rate, R^*, falls, this is expansionary at home.

- Case III: Flexible exchange rate § Keynesian supply. An increase in government spending shifts out the IS curve which causes the exchange rate to appreciate as the higher interest rates attract foreign investment which bids up the price of currency. Therefore IS shifts back in. Monetary policy is potent because the IS curve just follows shifts in LM which will increase output. Overall,

  $$\varphi = 0.$$

  $$\mu > 0.$$

  Therefore the IS curve is endogenous because the exchange appreciates for example following a fiscal stimulus and IS shifts right back. So only consider LM and AS when solving. (y, p, e) are the endogenous variables. When the world interest rate, R^*, falls, this is contractionary at home.

- Case IV: Flexible exhange rate § Classical supply. An increase in governement spending shifts out the IS curve which causes the exchange rate to appreciate as the higher interest rates attract foreign investment which bids up the price of currency. Domestic consumption falls as people are looking to buy foreign goods because of their relatively strong currency. Exports also fall because foreigners cannot afford goods at the higher exchange rate. Therefore IS shifts back in a little. The overall shift in IS causes prices to fall. Therefore real wages rise and labor supply therefore rises. Thus AS rises. LM will also shift out in (y, R) space because of the lower price level, real money balances rise. Monetary policy is impotent. Overall,

  $$\varphi > 0.$$

  $$\mu = 0.$$

  Therefore the IS curve is endogenous because the exchange appreciates for example following a fiscal stimulus and IS shifts right back. So only consider IS and AS when solving. (See Bean Notes). Fiscal policy note is still effective but only through the supply side. If the there is a fiscal push, IS out and back, but the appreciation of the exchange rate causes the labor supply to rise because real wage rises because real wage is a function of both domestic and foreign prices. (y, p, e) are the endogenous variables. When the world interest rate, R^*, falls, this is expansionary at home.

- In matrix form, cases I and II:

$$\begin{bmatrix} \alpha^* & \frac{s}{P}(c_s + x_s) & 0 \\ m_y & \frac{m}{P} & -\frac{1}{P} \\ 1 & -y_k' \; or \; \frac{s}{P}y_c' & 0 \end{bmatrix} \begin{bmatrix} dy \\ dP \\ dM \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} dg. \tag{8}$$

- Solving the system could be done with just a $2x2$ matrix instead of the $3x3$ because IS and AS do not depend on dM so this system is recursive. Indeed, after using cramers rule, $\varphi > 0$ and $\mu = 0$.

- For cases III and IV, the system is slightly more complicated,

$$\begin{bmatrix} \alpha^* & \frac{s}{P}(c_s + x_s) & \frac{P^*}{P}(x_s + c_s) \\ m_y & \frac{m}{P} & 0 \\ 1 & -y_k' \; or \; \frac{s}{P}y_c' & 0 \; or \; -y_c'\frac{P^*}{P} \end{bmatrix} \begin{bmatrix} dy \\ dP \\ de \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{P} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} dg \\ dM \end{bmatrix} \tag{9}$$

- Again, if the supply is Keynesian, the system is recursive so it can be reduced to a 2x2 matrix. In case IV however, the system of 3 equations and 3 unknowns must be solved.

5.1 The Effectiveness of Fiscal and Monetary Policy

- Consider a stochastic setting where we don't know the exact effects of policy. We must consider the policy measures independently, but also together because it may be possible to hedge the risk of one policy being too effective or not effective at all.

- In Poole's paper in 1970, he considered two different methods of policy actions. Policy makers could either target the money stock or the interest rate.

- If we target the interest rate, [**G-5.2**] the money stock become endogeneous because if there is a fiscal shock, the central bank will simply follow the shock with their own shift so the interest rate remains fixed.

- So which variable to target? Poole reasoned that it depended on the variation in the IS and LM curves, or the real and monetary sectors of the economy. If σ_{LM}^2 is relatively high, then target the interest rate. If σ_{IS}^2 is relatively high, then target the money stock. For example in the latter case: suppose there is a negative fiscal shock shifting IS in. If we were targeting the interest rate, the monetary athorities would have to shift the LM curve back as well to maintain a fixed R, thus reducing output even more. If the money stock was targeted, the reduction in output would be less.

- Consider the follow simple model of the economy:

$$IS : y = -aR + u,$$

where,

$$u \sim (0, \sigma_u^2).$$

$$LM : M = y - bR + v,$$

where,

$$v \sim (0, \sigma_v^2).$$

- Suppose we fix the Money Stock at $\bar{M}$.
  - Thus, solving for R in the LM equation,

    $$R = \frac{1}{b}[y - M + v].$$

    Substituting into the IS equation,

    $$y = -\frac{a}{b}[y - M + v] + u.$$

    And solving for y,

    $$y = \frac{a}{a+b}[M - v] + \frac{b}{a+b}u.$$

    Thus,

    $$\sigma_y^2 = \frac{a^2}{(a+b)^2}\sigma_v^2 + \frac{b^2}{(a+b)^2}\sigma_u^2 - \frac{2ab}{(a+b)^2}\sigma_{uv}.$$

- Suppose we fix the Interest Rate at $\bar{R}$.
  - If R is fixed, the only variability in y comes from u,

    $$\sigma_y^2 = \sigma_u^2.$$

5.2 The Brainard Model of Structural Uncertainty

- First denote,

  $$a = \varphi = \frac{dy}{dg} = \frac{1}{\alpha + \beta}.$$

  $$b = \mu = \frac{dy}{dM} = \frac{i_r}{m_R P}\varphi.$$

  Neglecting wealth effects.

- Suppose we are not sure about the values of these parameters, but we want to target some bliss level in the economy, y^*. We model the economy in the following way,

  $$y = ag + bM + u,$$

  Where a, b, and u are all stochastic components and $a \sim (\bar{a}, \sigma_a^2)$, $b \sim (\bar{b}, \sigma_b^2)$, $u \sim (0, \sigma_u^2)$, $Cov(a, b) = \sigma_{ab}$, and $Cov(a, u) = Cov(b, u) = 0$.

- Taking expectations,
$$E[y] = E[ag + bM + u] = \bar{a}g + \bar{b}M.$$
Our objective is to choose some combination of policy instruments to minimize,
$$E[(y - y^*)^2].$$

- **[G-5.3]** If we knew at least one of the policy measures, a or b, then we could obtain y^* exactly by just using only that policy measure and leaving nothing to chance. Mostly likely, both a and b will be unknown, so we would choose some combination of both that would not necessarily lead to y^* in expectation, but given the possible variablility of the two measures, we would choose to be on the safe side rather than have the measures to be too effective.

- Key to the analysis is σ_{ab}. If $\sigma_{ab} > 0$, then it is likely that if one measure is overestimating its effectiveness, the other is as well. So reduce (relatively) both measures: be more cautious. If $\sigma_{ab} < 0$, the effects of the policy measures will be offsetting so you can be a little more risky in choosing your expectation point. (Closer to y^*). If $\rho = \dfrac{\sigma_{ab}}{\sigma_a \sigma_b} = -1$, the two policy instruments are perfectly negatively correlated so you could actually choose each measure in expectation equal to y^*. It's a perfect hedge so you should end up right at the bliss point.

- The algebra of the Brainard model:

$$\begin{aligned}
E[(y - y^*)^2] &= E\Big[[(y - \bar{y}) + (\bar{y} - y^*)]^2\Big]. \\
&= E\Big[(y - \bar{y})^2 + 2(y - \bar{y})(\bar{y} - y^*) + (\bar{y} - y^*)^2\Big]. \\
&= \underbrace{E[(y - \bar{y})^2]}_{\sigma_y^2} + \underbrace{2E[(y - \bar{y})](\bar{y} - y^*)}_{0} + (\bar{y} - y^*)^2. \\
&= \underbrace{g^2\sigma_a^2 + M^2\sigma_b^2 + 2gM\sigma_{ab} + \sigma_u^2}_{\sigma_y^2} + (\underbrace{\bar{a}g + \bar{b}M}_{\bar{y}} - y^*)^2
\end{aligned}$$

- First Order Conditions:
$$\frac{\partial E[(y - y^*)^2]}{\partial g} \Rightarrow 2g\sigma_a^2 + 2M\sigma_{ab} + 2\bar{a}[\bar{a}g + \bar{b}M - y^*] = 0.$$
$$\frac{\partial E[(y - y^*)^2]}{\partial M} \Rightarrow 2M\sigma_b^2 + 2g\sigma_{ab} + 2\bar{b}[\bar{a}g + \bar{b}M - y^*] = 0.$$
Or written as a matrix,
$$\begin{bmatrix} \sigma_a^2 + \bar{a}^2 & \sigma_{ab} + \bar{a}\bar{b} \\ \sigma_{ab} + \bar{a}\bar{b} & \sigma_b^2 + \bar{b}^2 \end{bmatrix} \begin{bmatrix} g \\ M \end{bmatrix} = \begin{bmatrix} \bar{a} \\ \bar{b} \end{bmatrix} y^*. \tag{10}$$

- See case studies in the notes on 3.3 - 3.4.

6 Week 6 - Start of Jackman Lectures

6.1 Nominal Rigidities: New Classical Macroeconomics

- Business Cycles - the word cycle is slightly misleading in that the cycles are not in the periodic sense. They do however exhibit similar behavior across economic agents and thus we have several stylized facts about business cycles.

  - 1) Persistence or serial correlation. Periods of high (or low) activity tend to last for some time before slowing tailing off and the opposite cycle starts.
  - 2) Mutual correlation of macro variables. You don't have some variables flourishing and some decanting.
  - 3) $Cov(y, \dot{P}) > 0$. Output and inflation move together, ie, the Phillips curve: $\dot{P} = f(y - \bar{y}), f' > 0$.
  - 4) Stability of Real Wages.
  - 5) Procyclicality of Productivity, output per capita, $\frac{Y}{L}$.
  - Facts 4 and 5 are a problem because of the diminishing marginal product of labor. If we assume imperfect information, the facts are reasonable.
  - The Phillips curve relationship is a weakness. There really is no theoretical basis for the relationship and empirical evidence is weak at best.
  - The epithet "equilibrium" is usually used to describe the New Classical Macroeconomic models of the second half of the seventies together with the Real Business Cycle models of the eighties. However these approaches are generally characterised by "Market Clearing" models in perfectly competitive environments.

- Market Clearing Model of Aggregate Supply. In the normal labor market model,

$$L^d : \frac{W}{P} = F_L(L, k),$$

and,

$$L^s : L = l(\frac{W}{P}).$$

- The key way of allowing macroeconomic variables to enter into a market clearing model is imperfect information. We want a demand driven model so the only way to do this is through the price level. If workers do not have perfect information about the price level in the economy and hence about their real wages, we have to introduce price expectations. It is price surprises rather than the price level itself that matters. Define,

$$L^s = l(\frac{W}{P^e}) = l(\frac{W}{P} * \frac{P}{P^e}).$$

$$L = g(\frac{P}{P^e}; k).$$

Thus if we totally differentiate the Labor Supply equation, we get,

$$\begin{aligned}
dL &= l'\left\{\frac{P}{P^e}dF_L + F_L d(\frac{P}{P^e})\right\}. \\
&= l'\left\{\frac{P}{P^e}[\frac{\partial F_L}{\partial L}dL + \frac{\partial F_L}{\partial k}dk] + F_L d(\frac{P}{P^e})\right\}. \\
&= l'\left\{\frac{P}{P^e}[F_{LL}dL + F_{Lk}dk] + F_L d(\frac{P}{P^e})\right\}. \\
&= l'\frac{P}{P^e}F_{LL}dL + l'\frac{P}{P^e}F_{Lk}dk + l'F_L d(\frac{P}{P^e}). \\
dL - l'\frac{P}{P^e}F_{LL}dL &= l'\frac{P}{P^e}F_{Lk}dk + l'F_L d(\frac{P}{P^e}). \\
dL &= \frac{l'\frac{P}{P^e}F_{Lk}dk + l'F_L d(\frac{P}{P^e})}{1 - l'\frac{P}{P^e}F_{LL}}.
\end{aligned}$$

Now let $P = P^e$,

$$dL = \frac{l'F_{Lk}dk + l'F_L d(\frac{P}{P^e})}{1 - l'F_{LL}}.$$

So, the first partial of g, setting $dk = 0$,

$$g_1 = \frac{dL}{d\frac{P}{P^e}} = \frac{l'F_L}{1 - l'F_{LL}} > 0.$$

And, the second partial of g, setting $d\frac{P}{P^e} = 0$,

$$g_2 = \frac{dL}{dk} = \frac{l'F_{Lk}}{1 - l'F_{LL}} > 0.$$

Thus substituting back in the supply function,

$$y = F(L;k) = F(g(\frac{P}{P^e};k);k) = y_s(\frac{P}{P^e};k).$$

With,

$$y_{s1} = \frac{dy}{d\frac{P}{P^e}} = F_L g_1 > 0.$$

$$y_{s2} = \frac{dy}{dk} = F_L g_2 + F_k > 0.$$

- Note that unlike the Keynesian model in which the slope of the aggregate supply curve in (y, P) space depends only on the technology, here it also depends on the characteristics of the labour supply.

- To get significant fluctuation in employment (ie, stylized fact 5), we need a rather flat labour supply schedule so when there is a pricing surprise, employment moves dramatically.

- **[G-6.1]** If $\frac{P}{P^e} > 1$, or price expectations are low, the labor supply curve will shift to the right. If $\frac{P}{P^e} < 1$, or price expectations are higher than actual prices, the labor supply curve will shift to the left.

- As aggregate demand shifts out for whatever reason, prices begin to rise. Thus $\frac{P}{P^e} > 1$ which causes the labor supply to shift to the right, real wages fall and employment rises. People miss interpret the rise in wages because the price level is actually rising so real wages are falling.

6.2 The Lucas Islands Model

- The "Island" is a spacial analogue meaning that information is localized and particular to individual regions or islands.

- Each producer's supply function is defined as:

$$y_i = \bar{y}_i + \alpha(P_i - E_i[P]).$$

  Where y_i is firm i's output. $\bar{y}_i$ is the full information output level and $E_i[P]$ is firm i's expectation about the economy wide price level. If $P_i > E_i[P]$, then this is a sign that your firm or island should increase output to bring prices back down in line with the economy wide price level.

- Also,

$$E_i[P] = (1 - \theta)P_i + \theta P_0.$$

  Where P_i is the island price level and,

$$P_i = P + u_i \text{ where } u_i \sim N(0, \sigma_u^2).$$

  And P_0 is the unconditional expectation of the general price level given past data but with no current knowledge. Thus

$$P = P_0 + v \text{ where } v \sim N(0, \sigma_v^2).$$

  Thus u_i are sector or island specific shocks and v are economy wide shocks.

- It can be shown that,

$$\theta = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_v^2}.$$

- Substituting the $E[P]$ into the supply functions,

$$y_i = \bar{y}_i + \alpha(P_i - ((1 - \theta)P_i + \theta P_0)).$$

$$y_i = \bar{y}_i + \alpha(\theta P_i - \theta P_0).$$

$$y_i = \bar{y}_i + \alpha\theta(P_i - P_0).$$

- And now aggregating over all sectors or islands,

$$\sum_i y_i = y = \bar{y} + \alpha\theta(P - P_0).$$

- Now, we'll change some notation to match with the coming models: Let $P_0 = E_{t-1}[P] = P^e$. Thus,

$$y = \bar{y} + \alpha\theta(P - P^e).$$

So output fluctuations only occur if $P \neq P^e$ or people's price expectations differ from the actual price level. Thus this supply function is often called a "Surprise Function." Graphically, the short run aggregate supply is upward sloping for a given P^e, whereas the long run aggregate supply curve is vertical at $\bar{y}$ because $P = P^e$.

6.3 Rational Expectations and Determinants

- Expectations should surely be based on past experiences, because what else is there to base them on? Because of this, the term "Adaptive Expectations" is often used. However, expectations are fairly arbitrary because how do we know how the past is related to the present. (ie, should $E_{t+1}[P] = P_t$?). This type of analysis is also inefficient and a better method would make use of all information.

- So we now look to form expectations on the basis of some forecasting model. Consider,

$$X_t = \beta_0 + \beta_i y_{t-1}^i + \epsilon_t.$$

So we are trying to forecase X_t at time t given some set of history variables y^i at time $t-1$. NOTE that we cannot forecast X_t using y_t. There must exist a timing gap in the model.

- We then go ahead and estimate this model using the usual methods and find estimates for the parameters, $(\hat{\beta}_0, \hat{\beta}_i)$.

- Thus,

$$E[X_t] = \hat{\beta}_0 + \hat{\beta}_i y_{t-1}^i.$$

And make the usual assumptions about ϵ_t. Also,

$$X_t - E[X_t] = \epsilon_t,$$

by definition.

- When we refer to the term "Rational Expectations" we are referring to a situation when expectations are "model consistent." In other words, the people that the expectations forecasting model is based on have expectations as if they already knew the model that we are using to forecast. As if the people use our model to determine their expectations.

- Define a simply aggregate demand / aggregate supply model as follows:

$$AD : m_t - P_t = \beta y_t^d + v_t.$$

$$AS : y_t^s = \bar{y} + \alpha(P_t - E_{t-1}[P_t]) + u_t.$$

  In the AD equation, all variables: m, P, and y are in logarithims so on the left hand side there is the real money balances and on the right is aggregate demand plus shocks. In the AS equation, we have the model just developed previously.

- Setting $y_t^s = y_t^d$ implies,

$$\bar{y} + \alpha(P_t - E_{t-1}[P_t]) + u_t = \frac{m_t - P_t - v_t}{\beta}.$$

  And solving for P_t,

$$P_t = \frac{m_t + \alpha\beta E_{t-1}[P_t] - \beta\bar{y} - \beta u_t - v_t}{1 + \alpha\beta}.*$$

  If we assume that the shocks to AS and AD are normally distributed with mean 0 and variance σ^2,

$$E_{t-1}[P] = \frac{E_{t-1}[m_t] + \alpha\beta E_{t-1}[P_t] - \beta\bar{y}}{1 + \alpha\beta}.**$$

  And now solving for $E_{t-1}[P]$,

$$E_{t-1}[P] - \frac{\alpha\beta E_{t-1}[P]}{1 + \alpha\beta} = \frac{E_{t-1}[m_t] - \beta\bar{y}}{1 + \alpha\beta}.$$

$$E_{t-1}[P](1 - \frac{\alpha\beta}{1 + \alpha\beta}) = \frac{E_{t-1}[m_t] - \beta\bar{y}}{1 + \alpha\beta}.$$

$$E_{t-1}[P](\frac{1 + \alpha\beta - \alpha\beta}{1 + \alpha\beta}) = \frac{E_{t-1}[m_t] - \beta\bar{y}}{1 + \alpha\beta}.$$

$$E_{t-1}[P](\frac{1}{1 + \alpha\beta}) = \frac{E_{t-1}[m_t] - \beta\bar{y}}{1 + \alpha\beta}.$$

$$E_{t-1}[P] = E_{t-1}[m_t] - \beta\bar{y}.$$

  Thus, to find the differential in prices and price expectations, consider the difference between equation $*$ and $**$,

$$P_t - E_{t-1}[P] = \frac{m_t - E_{t-1}[m_t] - \beta u_t - v_t}{1 + \alpha\beta}.$$

- Thus only unexpected shocks in the money stock or fluctuations in the supply or demand shocks will change output.

- Substituting $P_t - E_{t-1}[P_t]$ into the AS equation yields,

$$AS : y_t^s = \bar{y} + \alpha(P_t - E_{t-1}[P_t]) + u_t.$$

$$y_t^s = \bar{y} + \alpha\Big(\frac{m_t - E_{t-1}[m_t] - \beta u_t - v_t}{1 + \alpha\beta}\Big) + u_t.$$

$$y_t^s = \bar{y} + \frac{\alpha}{1 + \alpha\beta}\Big[m_t - E_{t-1}[m_t] - \beta u_t - v_t\Big] + u_t.$$

- We now need to address the issue of $E_{t-1}[m_t]$ or the expection of monetary policy in the economy. Suppose there exists a monetary rule such that,

$$m_t = m_0 + g_t - \gamma y_{t-1} + \epsilon_t.$$

  Where m_0 is some fixed supply, g_t is a growth of money term, and the γ term multiplied by the lagged output is sort of a stabilization factor. If output is too high in the pervious period, γ will revise the current period money supply down. Finally, ϵ_t are unpredictable effects but $\epsilon_t \sim N(0, \sigma^2)$.

- Thus

$$E_{t-1}[m_t] = m_0 + g_t - \gamma y_{t-1}.$$

  And,

$$m_t - E_{t-1}[m_t] = \epsilon_t.$$

  Which can be thought of as the "unanticipated money."

- Substituting the unanticipated money into the AS equation above,

$$y_t^s = \bar{y} + \frac{\alpha}{1 + \alpha\beta}\Bigg[\epsilon_t - \beta u_t - v_t\Bigg] + u_t.$$

- Thus any systematic component of monetary policy has no effect on output at any point in time. People are able to predict what is going to happen so only the surprises, (ϵ_t) are effective. Hence the "Policy Ineffectiveness Propostition" or PIP. However $E[\epsilon_t] = 0$ so even unsystematic policy can effect y_t only in the short term, but can never effect $\bar{y}$, or the long term aggregate output. Also, if $Cov(\epsilon_t, u_t) = Cov(\epsilon_t, v_t) = 0$ then the unsystematic policy has no spillover effects on the other shock terms but may increase the variance of those terms (u and v). However, if one of the previous covariances is positive then the unsystematic policy might be effective.

- Thus only shocks to demand and supply can drive output away from its natural rate. The immediate corollary is that no feedback policy can affect output since $(m_t - E_{t-1}[m_t])$ is by construction uncorrelated with all the past information to which the authorites are responding. For policy to be effective, it requires that the government have some sort of informational advantage over the private sector. However, at the same time, if they had this information, wouldn't it be best just to release that information and let the private sector respond how it wanted to and not try to manipulate it? This story leads to the conclusion that the authorities should follow a systematic rule, simply trying to avoid introducing further variability into output through the unanticipated component of the money stock.

7 Week 7

7.1 Monetary Instruments

- Thus far under the heading of the "New Classical Macro," we have considered the classical case where prices are flexible and markets clear. This analysis gave us the policy ineffectiveness principal. We have also studied incomplete information and rational expectations.
- Now consider the two different monetary instruments: the interest rate, R, and real money balances, m.
- Under interest rate targeting, $R_t = \bar{R}$.
  -
    $$(IS) : y_t^d = a - bR_t + v_{1t}$$
    $$(AS) : y_t^s = \bar{y} + \alpha(p_t - E_{t-1}[p_t]) + u_t.$$
    Setting $y_t = y_t^d = y_t^s$,
    $$p_t = E_{t-1}[p_t] + \frac{1}{\alpha}\Big(a - b\bar{R} + v_{1t} - \bar{y} - u_t\Big).$$
    Thus,
    $$E_{t-1}[p_t] = E_{t-1}[p_t] + \frac{1}{\alpha}\Big(a - b\bar{R} - \bar{y}\Big).$$
    But we can't solve this equation for price expectations so there is no basis for forming those expectations based on this model. The aggregate demand curve is invariate with respect to prices.
  - Recall from the last model,
    $$(AD) : m_t - p_t = \beta y_t^d + v_t.$$
    $$(AS) : y_t^s = \bar{y} + \alpha(p_t - E_{t-1}[p_t]) + u_t.$$
    Taking expectations,
    $$(AD) : E[m_t] - E[p_t] = \beta E[y_t^d].$$
    $$(AS) : E[y_t^s] = \bar{y} + \alpha(E_{t-1}[p_t] - E_{t-1}[p_t]) = \bar{y}.$$
    Setting $E[y_t^d] = E[y_t^s]$,
    $$E[p_t] = E[m_t] - \beta\bar{y}.$$
  - But now in our model, if we take expectations of supply and demand, we get:
    $$(IS) : E[y_t^d] = a - b\bar{R}$$
    $$(AS) : E[y_t^s] = \bar{y}.$$
    So both curves are invariate with respect to prices so it would seem that a policy based on interest rates would be ineffective. So why do central banks still use interest rate targeting as opposed to real money balance targeting? There seems to be a dichotomy between actions and economic theory.

– Actually, monetary authorities use something called a "Contingent Interest Rate Rule."

7.2 The Contingent Interest Rate Rule

– Define:

$$R_t = \bar{R} + \mu(E_{t-1}[p_t] - p_t^*).$$

Hence μ represents the deviation of price level from some target level. ie, if $E_{t-1}[p_t] > p_t^*$, then raise interest rates.

– Thus substituting into our IS and AS equations,

$$(IS) : y_t^d = a - b(\bar{R} + \mu(E_{t-1}[p_t] - p_t^*)) + v_{1t}$$
$$(AS) : y_t^s = \bar{y} + \alpha(p_t - E_{t-1}[p_t]) + u_t.$$

Taking Expectations,

$$E[y_t^d] = a - b\bar{R} - b\mu(E_{t-1}[p_t] - p_t^*)$$
$$E[y_t^s] = \bar{y}.$$

Setting $E[y_t^d] = E[y_t^s]$,

$$\bar{y} = a - b\bar{R} - b\mu(E_{t-1}[p_t] - p_t^*).$$

Solving for $E_{t-1}[p_t]$,

$$E_{t-1}[p_t] = \frac{1}{b\mu}\big(a - b\bar{R} + b\mu p^* - \bar{y}\big).$$

$$E_{t-1}[p_t] = p^* + \frac{1}{b\mu}\big(a - b\bar{R} - \bar{y}\big).$$

This is the KEY EQUATION in all this bullshit. Note that price expectations can now be determined. Subtracting p^* from both sides,

$$E_{t-1}[p_t] - p^* = \frac{1}{b\mu}\big(a - b\bar{R} - \bar{y}\big).$$

Substituting this back into R_t equation,

$$R_t = \bar{R} + \mu\big(\frac{1}{b\mu}(a - b\bar{R} - \bar{y})\big) = \bar{R} + \frac{1}{b}\big(a - b\bar{R} - \bar{y}\big) = \frac{a}{b} - \frac{\bar{y}}{b}.$$

Substituting back in the equation for y_t^d,

$$(IS) : y_t^d = a - b(\bar{R} + \mu(E_{t-1}[p_t] - p_t^*)) + v_{1t}$$
$$y_t^d = a - b\bar{R} - b\mu\left(\frac{1}{b\mu}(a - b\bar{R} - \bar{y})\right) + v_{1t}$$
$$y_t^d = a - b\bar{R} - (a - b\bar{R} - \bar{y}) + v_{1t}$$
$$y_t^d = \bar{y} + v_{1t}.$$

Thus aggregate demand shocks are all that will shift AD.

- Some information on real money balance targeting:
  - m_t is now exogeneous and R_t is determined in the market. Must distinguish real from nominal interest rates:
$$r_t = R_t - E_{t-1}[p_{t+1} - p_t] = R_t - \dot{p}.$$
  - Suppose authorities base money supply on past prices such that,
$$m_{t+1} = \gamma_0 - \gamma_1 p_t.$$
    But everyone could predict this rule and policy would be ineffective. If p_t, current prices, are too high, m_{t+1} would be lower, and prices tomorrow would then be lower. But in our setting, price expectations lead to adjustments to output so,
$$r_t = R_t - E_t[p_{t+1} - p_t].$$
$$y_t^s = \alpha(p_t - E_{t-1}[p_t]) + u_t.$$
    Note the expectations of inflation are taken at time t versus the supply equation expectations occur at time $t-1$. This was not explained very well in lecture but this difference is imporant. The difference in expectations creates an asymmetry in information that gets around the policy ineffectiveness principal. Since the financial markets somehow have more information when determining price expectations but the government and producers are at an informational disadvantage, this makes policy measures effective (??).

7.3 Testing the New Classical Macro

- Define price expectations,
$$p_t^e = E_{t-1}[p_t],$$
  and,
$$\epsilon_t = p_t - p_t^e.$$
  Thus we would expect $E[\epsilon_t] = 0$ and $cov(\epsilon_t, \epsilon_{t-1}) = 0$. The first expectation is relatively consistant over long periods of time but the serial uncorrelation is harder to find.
- The problem with the new classical macro is the stylized business cycle fact number 1: Persistence. We tend to see that output in one period is correlated with previous output levels. But consider the supply equation in our model:
$$AS : y_t^s = \bar{y} + \alpha(p_t - E_{t-1}[p_t]) + u_t.$$
  Note that there is no y_{t-1} term on the right side which, because of rational expectations, yields serial uncorrelation of output levels. To account for what we see in reality, introduce a definition of $\bar{y}$,
$$\bar{y}_t = \lambda\bar{y}_0 + (1-\lambda)y_{t-1}.$$
  This would clearly give us serial correlation. A story one could tell is that if unemployment is high in period t, skills will begin to deteriorate among workers so that in period $t+1$, production will be lower.

- Substituting this equation into the supply equation,

$$AS : y_t^s = \lambda \bar{y}_0 + (1-\lambda)y_{t-1} + \alpha(p_t - E_{t-1}[p_t]) + u_t.$$

7.4 Testing the Policy Ineffectiveness Principal

- Consider the following AD and AS equations,

$$AD : m_t - p_t = \beta y_y^d + v_t.$$

$$AS : y_t^s = \alpha(p_t - E_{t-1}[p_t]) + u_t.$$

Solving the system as was done earlier,

$$y_t = \frac{1}{1+\alpha\beta}\Bigg[\alpha(m_t - E_{t-1}[m_t]) + u_t - \alpha v_t\Bigg].$$

And simplifying the expression for no apparent reason ...

$$y_t = \lambda(m_t - E_{t-1}[m_t]) + \eta_t.$$

- Now consider the monetary rule from earlier,

$$m_t = m_0 + g_t + \gamma y_{t-1} + \epsilon_t.$$

And taking expectations,

$$E_{t-1}[m_t] = m_0 + g_t + \gamma y_{t-1}.$$

Thus,

$$m_t - E_{t-1}[m_t] = \epsilon_t.$$

- Substituting this equation into the supply equation above,

$$y_t = \lambda\epsilon_t + \eta_t.$$

Which shows again the PIP: output is independent of g and γ.

- Suppose we think that the money rule is the correct specification. Thus Anticipated Money is,

$$E_{t-1}[m_t] = \hat{m}_t = m_0 + g_t + \gamma y_{t-1}.$$

And Unanticipated Money is,

$$m_t - E_{t-1}[m_t] = \tilde{m}_t = \epsilon_t.$$

Thus,

$$m_t = \hat{m}_t + \tilde{m}_t.$$

To test the PIP, we could then run a regression of the form:

$$y_t = \lambda_0 + \lambda_1 \hat{m}_t + \lambda_2 \tilde{m}_t + \eta_t.$$

If PIP holds, $\lambda_1 = 0$ and $\lambda_2 > 0$, as unanticipated money is the only thing that matters. Regressions of this nature tell us absolutely nothing. Positive, negative, and insignificant ... any theory can be supported by a different analysis. So what the hell are we doing this for!!

- We can also consider Observational Equilvalence. Consider the following model:

  $$y_t = \lambda(m_t - E_{t-1}[m_t]) + \eta_t.$$

  Substituting in $E_{t-1}[m_t]$,

  $$y_t = \lambda m_t - \lambda[m_0 + g_t + \gamma y_{t-1}] + \eta_t.$$

  $$y_t = \lambda m_t - \lambda m_0 - \lambda g_t - \lambda\gamma y_{t-1} + \eta_t.$$

  Which is a reasonable Keynesian model. If output is low in one year and as a result, the central bank increases the money supply, if output does not rise as much as predicted, there are two explanations based on this model. Either 1) the lagged y_{t-1} term in the equation was larger than expected or 2) The anticipated versus unanticipated money story where people expect the money supply to rise so when it does, the effects are smaller. (PIP)

8 Week 8

8.1 The Lucas Critique

- Consider the following model of the economy:

$$AS : y_t = \alpha(p_t - p_t^e) + u_t.$$

$$AD : m_t - p_t = \beta y_t + v_t.$$

$$\text{Money Rule: } m_t = \gamma + p_{t-1}.$$

- Note that $E[y_t^s] = 0$ and $E_{t-1}[m_t] - E_{t-1}[p_t] = \beta E[y_t] = \beta * 0 = 0$. Thus,

$$E_{t-1}[m_t] = E_{t-1}[p_t].$$

- From the money rule, $E_{t-1}[m_t] = \gamma + p_{t-1}$, so

$$E_{t-1}[m_t] = E_{t-1}[p_t] = \gamma + p_{t-1}.$$

- Substituting AS into AD,

$$m_t - p_t = \beta\Big(\alpha(p_t - p_t^e) + u_t\Big) + v_t.$$

$$m_t - p_t = \alpha\beta p_t - \alpha\beta p_t^e + \beta u_t + v_t.$$

  Solving for p_t,

$$p_t + \alpha\beta p_t = m_t + \alpha\beta p_t^e - \beta u_t - v_t.$$

$$p_t = \frac{m_t + \alpha\beta p_t^e - \beta u_t - v_t}{1 + \alpha\beta}.$$

  Thus, taking expectations,

$$E_{t-1}p_t = p_t^e = \frac{E_{t-1}m_t + \alpha\beta p_t^e}{1 + \alpha\beta}.$$

$$p_t^e - \frac{\alpha\beta p_t^e}{1 + \alpha\beta} = \frac{E_{t-1}m_t}{1 + \alpha\beta}.$$

$$p_t^e(1 - \frac{\alpha\beta}{1 + \alpha\beta}) = \frac{E_{t-1}m_t}{1 + \alpha\beta}.$$

$$p_t^e(\frac{1 + \alpha\beta - \alpha\beta}{1 + \alpha\beta}) = \frac{E_{t-1}m_t}{1 + \alpha\beta}.$$

$$p_t^e(\frac{1}{1 + \alpha\beta}) = \frac{E_{t-1}m_t}{1 + \alpha\beta}.$$

$$p_t^e = E_{t-1}m_t.$$

  As was shown above by another method.

- What about the Phillips Curve? Since,

$$p_t^e = E_{t-1}m_t = \gamma + p_{t-1}.$$

Substituting this into the AS equation,

$$y_t = \alpha(p_t - \gamma - p_{t-1}) + u_t.$$

$$y_t = -\alpha\gamma + \alpha(p_t - p_{t-1}) + u_t.$$

$$y_t = -\alpha\gamma + \alpha\dot{p} + u_t.$$

- So in $(y, \dot{p})$ space [**G-8.1**], the phillips curve slopes upwards and intersects the vertical axis where $\dot{p} = \gamma$. Call this point γ_1. See Graph in notes.

- Now the government asks, given the phillips curve, is monetary policy optimal? If the government wants $y_t = 0$, or zero growth of output, then all they have to do is set $m_t = \gamma_1$.

- But now suppose the government wants to target some level of output growth, y^* so that

$$y_t = y^* \ \forall \ t,$$

and

$$p_t - p_{t-1} = \dot{p} = 0 \ \forall \ t.$$

- The target output level, y^*, is often referred to as the "bliss point."

- To determine how well a policy is working, define a loss function such that,

$$L = \lambda(y_t - y^*)^2 + (p_t - p_{t-1})^2.$$

Thus λ is just the relative weight placed on output deviations but in general, output and price deviations are equally bad for the economy and positive and negative deviations count equally in the loss function (via the square).

- Again this is seen better graphically, but the loss function looks like a series of "IsoLoss" circles centered around the bliss point, y^*. Clearly, γ_1 is not the best that the government can do in minimizing the loss function though if they take the phillips curve as given, they can never reach the bliss point. So a point on the Phillips curve which is perpendicular to a line that connects the point to y^* is the best the government can do [**G-8.1**].

- Or can they? It appears that if the government alters γ sufficiently, they could achieve this point. However, when γ is altered to say γ_2, the phillips curve shifts up so that it intersects the $\dot{p}$ axis at γ_2. The economy ends up much worse off then when it started.

- Thus this method fails when looking at observed econometric relationships to determine policy rule because econometric relationships enter into the reality of the phillips curve.

- If this continued and the government kept shifting γ to reach this optimal point, we would eventually reach γ^* where the isoloss curve is just tangent to the phillips curve at the "optimal" level of γ. **[G-8.2]**
- Note that the Lucas critique does not rely on rational expectations. Merely the endogeneity of expectations formation and other behavioural rules. Empirical relevance of the Lucas Critique has yet to be vertified.
- Another famous quote about Bob Lucas: "The Lucas program of specifying models in terms of their 'deep' structural parameters lies at the heart of contemporary macroeconomics." - Lucas 88a.

8.2 New Keynesian Macroeconomics

- The difference between New Keynesian and New Classical is that New Keynesian does not assume market clearing and prices and wages are not perfectly flexible (Price/Wage Stickiness).
- The labor market also does not instantaneously clear. It might take a while for wages to adjust.
- These price and wage inflexibilities are called Nominal Rigidities.
- The "New" part of "New Keynesian" Macroeconomics is that we attempt to explain where the rigidities come from. We explain this using micro foundations. We will also assume Rational Expectations.
- The Basic Model.
  - Wage contract is written in nominal terms for a given duration of time (usually long). The contract is not adjusted every day because renegotiation is costly.
  - Workers, even though they negotiate nominal wages, are really only interested in real wages. Define the target real wage for period t as ω_t. But because of long term contracts, nominal wage is fixed at,
  $$\bar{W}_t = \omega_t E_{t-1}[p_t] = \omega_t p_t^e.$$
  Note that real wages equal nominal wages divided by expected prices: $\omega_t = \frac{\bar{W}_t}{p_t^e}$.
  - Employment is determined by labor demand via the FOC,
  $$\frac{\bar{W}_t}{p_t} = F_L(l,k).$$
  Substituting in for nominal wages,
  $$\frac{\omega_t p_t^e}{p_t} = F_L(l,k).$$
  - Thus, if inflation is higher than expected, $p_t > p_t^e$ and real wages therefore fall (because contracts cannot be altered immediately) and employment rises. (See graph in notes).

8.3 More on the New Keynesian Macro

- There are three main types of Keynesian labor market models. 1) Old fashioned Keynesian models where there is always unemployment in the economy because labor supply is always greater than labor demand. 2) Non-market clearing of the Real Wage models: unions determine the optimal real wages and labor supplied based on some set of union indifference curves but of course price fluctuations move us away from this level of real wage. 3) Frictional Unemployment Models: where there is always some degree of unemployment (ie, the Natural Rate of Unemployment) which corresponds to an employment curve to the left of labor supply. [**G-8.3**] All these models give us results similar to the classical case.

- Consider Labor Demand:
$$l_t = -\lambda(w_t - p_t).$$
Where all terms are in logs so w_t (the nominal wage) $= \omega_t$ (the real wage) $+p_t^e$ (expected prices). By choice of units we can always set $\omega_t = 0$ (normalised to 1 because of the log transformation). Thus,
$$w_t = 0 + p_t^e = p_t^e.$$
Substituting in,
$$l_t = -\lambda(p_t^e - p_t) = \lambda(p_t - p_t^e).$$

- Now consider the production function as follows:
$$y_t = \phi l_t + u_t.$$
Where ϕ is a technological factor. Substituting in for l_t,
$$y_t = \phi\lambda(p_t - p_t^e) + u_t.$$
Which is precisely the same as the new classical supply except now the factors are technological whereas before they were behaviorial. Simplifying,
$$y_t = \alpha(p_t - p_t^e) + u_t.$$

- Now we would like to ask: what if the duration of wage contracts is longer than it takes for information to flow through the economy?

- Consider two groups of labors/unions/etc. Group A and Group B. Group A negotiates contracts in period t and the contract is in effect until period $t + 2$ when it then renegotiated. In period t, group B is in the middle of their contract and in period $t+1$ they negotiate new ones. Their contracts also last 2 periods (until $t + 3$) when they too renegotiate for a further two periods.

- Note that if contracts were adjusted EVERY period, this sort of model will produce the usual policy ineffectiveness result under rational expectations.

- Today is Ariel's Birthday. [**G-8.4**]

- Since workers or unions negotiate nominal wages but are only interested in real wages, they have to make expectations about the price level in the coming two periods. Thus, in period $t+2$,

$$\text{Groups A Sets } W^A_{t+2} \text{ based on } E_{t+1}[p_{t+2}].$$

$$\text{Groups A Sets } W^A_{t+3} \text{ based on } E_{t+1}[p_{t+3}].$$

- Similarly, in period $t+1$,

$$\text{Groups B Sets } W^B_{t+1} \text{ based on } E_t[p_{t+1}].$$

$$\text{Groups B Sets } W^B_{t+2} \text{ based on } E_t[p_{t+2}].$$

  Because both groups only have knowledge up until the period immediately prior to when they are making expectations.

- So in period $t+2$, somes wages are based on $E_t[p_{t+2}]$ and some are based on $E_{t+1}[p_{t+2}]$, noting the difference in the timing of the expectations.

- So what is the economy wide wage. Ignoring the fact that these values are in logs so the next calculation would not make sense, let the economy wide wage be the average of the two group's wages. Thus,

$$W^*_{t+2} = \frac{1}{2}W^A_{t+2} + \frac{1}{2}W^B_{t+2}.$$

  Note that this is the economy wide NOMINAL wage.

- Applying the definition of nominal wage: $w_t = \omega_t + p^e_t$ and ω_t is normalized to 0,

$$W^*_{t+2} = \frac{1}{2}E_{t+1}[p_{t+2}] + \frac{1}{2}E_t[p_{t+2}].$$

  Subtracting off p_{t+2},

$$W^*_{t+2} - p_{t+2} = \frac{1}{2}(E_{t+1}[p_{t+2}] - p_{t+2}) + \frac{1}{2}(E_t[p_{t+2}] - p_{t+2}).$$

  Since $l_t = -\lambda(W_t - p_t)$.

$$l_{t+2} = -\lambda\left(\frac{1}{2}(E_{t+1}[p_{t+2}] - p_{t+2}) + \frac{1}{2}(E_t[p_{t+2}] - p_{t+2})\right).$$

  Thus, substituting l_{t+2} into the supply function,

$$y_{t+2} = \phi\left[-\lambda\left(\frac{1}{2}(E_{t+1}[p_{t+2}] - p_{t+2}) + \frac{1}{2}(E_t[p_{t+2}] - p_{t+2})\right)\right] + u_t.$$

  Combining the coefficients and multiplying through the negative sign,

$$y_{t+2} = \frac{1}{2}\alpha(p_{t+2} - E_{t+1}[p_{t+2}]) + \frac{1}{2}\alpha(p_{t+2} - E_t[p_{t+2}]) + u_t.$$

- Now consider aggregate demand:

$$AD : m_t - p_t = \beta y_t + v_t.$$

- Under this simple setting, the same as in the classical model, any shocks to AD will only temporarly increase AD but since people will still be expecting AD to shift back to its orginal level (they have no reason to think differently), AS does not shift and we eventually end up back where we started. **[G-8.5]** Now in this setting, it makes sense to introduce some presistance of the demand shocks such that,

$$v_t = \rho v_{t-1} + \epsilon_t.$$

  So a demand shock in period t will result in AD shifting back a little in period $t+1$ but not all the way back.

- Note that in the classical case, adding in persistance did nothing to affect the variability of output because we always ended up right back at $\bar{y}$ due to the flexibility of wages and prices, so we left it out. In the Keynesian setting however, the persistance is an important characteristic.

- Taking expectations of AD:

$$AD : m_t - p_t = \beta y_t + v_t = \beta y_t + \rho v_{t-1} + \epsilon_t.$$

$$E[AD] : E_{t-1}[m_t] - E_{t-1}[p_t] = \beta E_{t-1}[y_t] + E_{t-1}[v_t] = \beta E_{t-1}[y_t] + \rho v_{t-1}.$$

$$AD - E[AD] = (m_t - p_t) - (E_{t-1}[m_t] - E_{t-1}[p_t]) = \beta(y_t - E_{t-1}[y_t]) + \epsilon_t.$$

- Taking expectations of AS (first rewritting AS):

$$AS : y_{t+2} = \frac{1}{2}\alpha(p_{t+2} - E_{t+1}[p_{t+2}]) + \frac{1}{2}\alpha(p_{t+2} - E_t[p_{t+2}]) + u_t.$$

$$y_{t+2} = \frac{1}{2}\alpha p_{t+2} - \frac{1}{2}\alpha E_{t+1}[p_{t+2}] + \frac{1}{2}\alpha p_{t+2} - \frac{1}{2}\alpha E_t[p_{t+2}] + u_t.$$

$$y_{t+2} = \alpha p_{t+2} - \frac{1}{2}\alpha E_{t+1}[p_{t+2}] - \frac{1}{2}\alpha E_t[p_{t+2}] + u_t.$$

$$y_{t+2} = \alpha p_{t+2} - \alpha E_{t+1}[p_{t+2}] + \frac{1}{2}\alpha E_{t+1}[p_{t+2}] - \frac{1}{2}\alpha E_t[p_{t+2}] + u_t.$$

$$y_{t+2} = \alpha\Big(p_{t+2} - E_{t+1}[p_{t+2}]\Big) + \frac{1}{2}\alpha\Big(E_{t+1}[p_{t+2}] - E_t[p_{t+2}]\Big) + u_t.$$

$$E[AS] : E_{t+1}[y_{t+2}] = \alpha\Big(E_{t+1}p_{t+2} - E_{t+1}[p_{t+2}]\Big) + \frac{1}{2}\alpha\Big(E_{t+1}[p_{t+2}] - E_t[p_{t+2}]\Big) + 0.$$

$$E_{t+1}[y_{t+2}] = \frac{1}{2}\alpha\Big(E_{t+1}[p_{t+2}] - E_t[p_{t+2}]\Big).$$

$$E_{t-1}[y_t] = \frac{1}{2}\alpha\Big(E_{t-1}[p_t] - E_{t-2}[p_t]\Big).$$

- For the next part, it seems that in the notes, he takes the formula above for $E[AD]$ and substitutes in $y_t = \alpha(p_t - p_t^e) + u_t$ and then takes it back one period with expectations. Then substitutes these equations into the original supply equation,

$$y_{t+2} = \alpha\Big(p_{t+2} - E_{t+1}[p_{t+2}]\Big) + \frac{1}{2}\alpha\Big(E_{t+1}[p_{t+2}] - E_t[p_{t+2}]\Big) + u_t.$$

  Which yields,

$$y_t = \frac{\alpha}{1+\alpha\beta}(m_t - E_{t-1}[m_t] - \beta u_t - \epsilon_t) + \frac{\alpha}{2+\alpha\beta}(E_{t-1}[m_t] - E_{t-2}[m_t] - \rho\epsilon_{t-1}) + u_t.$$

- Thus, the first term is a pure surprise term and authorities cannot exploit this. But by setting,

$$m_t = \rho\epsilon_{t-1},$$

  the second term is assured to be equal to exactly 0 and then they can minimize the variance of output. This is essentially a restatement of the traditional Keynesian argument for stabilisation policy, but in a rational expectations setting.

- Thus in an economy with lacking wage/price flexibility, the government has some role for policy though the effects only last one "informational period."

9 Week 9

9.1 New Keynesian Macro - Menu Cost Models

- The new Keynesian macro is based on long contracts in the labor market. One of the explanations for this is the high cost of renegotiation. However, a classical response to the new Keynesian macro is that it doesn't seem likely that these costs are so enormous as to cause the nominal rigidities that we see in the markets. For instance to say that the lack of negotiation in contracts caused the great depression would be craziness.

- So what is the cause of the rigidities when the costs of changing contracts is fairly small? We will show that even when these "Menu costs" are small for the firm, the overall macroeconomic impact is rather large and might lead to the nominal rigidities that Keynesian models are based upon.

- We will need to model markets with imperfect competition. Thus firms choose the prices that they will charge in the market and their output and labor decisions depend on this price. Firm profits as usual,

$$\pi_i = p_i y_i - w_i L_i.$$

Maximizing with respect to prices and setting equal to 0 will determine the firm's equilibrium price, and therefore output quanitity and labor choice.

$$\frac{\partial \pi}{\partial p_i} = 0.$$

- Thus when there is a shock to the economy that changes the economy wide price level, profits for individual firms should remain about constant because of this first order condition. In other words, profits are locally invariable to price changes.

- Consider the graph in the notes for the next step. [**G-9.1**] We consider a wage fall at the economy wide level. If prices for the firm remain the same, this immediately increases firm profits. If the firm was to reoptimize and lower their prices (see graph), they would make further profit gains, but would also lose out a bit because of the lower price. The net effect is small.

- Note that $\pi(P_i(W), P, W, y, k_i)$, so $\frac{d\pi}{dW} = \frac{\partial \pi}{\partial W} + \frac{\partial \pi}{\partial P}\frac{\partial P}{\partial W} = \frac{\partial \pi}{\partial W}$ (by the envelope theorem.)

- So it seems like it really doesn't matter if firms readjust (reoptimize) their prices or not when there is the shock.

- Consider the following model:

  - Firm level demand:

$$y_i^d = (\frac{P_i}{P})^{\epsilon} \frac{y}{n}.$$

  Where y is aggregate GDP and ϵ is the elasticity of demand.

- Firms have a production function,

$$y_i = L_i.$$

- Firms maximize,

$$\pi = (P_i - W)(\frac{P_i}{P})^{\epsilon}\frac{y}{n},$$

with respect to their own price level.
- Solving this optimization yields,

$$P_i = \frac{\epsilon}{\epsilon - 1}W.$$

- At the economy level, aggregate demand is just equal to the real money balance,

$$y = \frac{M}{P}.$$

- And aggregating over the firms, the economy price level is:

$$P = \frac{\epsilon}{\epsilon - 1}W.$$

Thus the real wage in the economy is,

$$\frac{W}{P} = \frac{\epsilon - 1}{\epsilon}.$$

- Labor supply in the economy is,

$$L^s = (\frac{W}{P})^{\delta},$$

with $\delta > 0$.
- Thus, again referring to the graph in the notes, in the labor market, the real wage is fixed at $\frac{\epsilon-1}{\epsilon}$, labor supply is upward sloping, and aggregate demand can be drawn vertical at the equilibrium level of employment such that,

$$L^* = y^* = \frac{M}{P}^*.$$

- Initally the money stock is at M_0. But now consider an adverse money shock so that $M_1 < M_0$. The immediate result is AD shifting to the left to $\frac{M_1}{P}$. If wage and prices remain constant, we simply move from E_0 to E_0'. But at this point, $L^S > L^D$. All this unemployment in the economy will drive the wage down and we'll end up at E_1 which is not profit maximizing for the firm. So the question now becomes what should the firm do? Renegotiate contracts and change the price structure or should they keep prices at the same level? The answer would be simple if it weren't for these extra menu costs that the firm would incurr if it decided to change prices.

– To determine the firm's decision, compare the profits from fixing the price as it was and the profits from adjusting the price to the new profit maximizing level. Consider REAL profits to reflect the change in prices.

– Profits from staying at old price:

$$\begin{aligned}\pi_i^F = \frac{P_i y_i - W L_i}{P} &= \frac{P_i y_i - W y_i}{P} \\ &= (P_i - W)(\frac{y_i}{P}) \\ &= (P - W)(\frac{y_i}{P}) \\ &= (1 - \frac{W}{P})(y_i) \\ &= (1 - \frac{W}{P})\frac{y}{n} \\ &= (1 - L^{1/\delta})\frac{y}{n} \\ &= (1 - (\frac{M_1}{P})^{1/\delta})(\frac{M_1}{P})\frac{1}{n}\end{aligned}$$

– And this quantity isn't profit maximizing none the less. Now consider what would happen if the firm did adjust prices. Now we calculate profits adding and additional cost term, c, to reflect the "menu costs." First note the new price level that is optimizing:

$$\begin{aligned}P_i = \frac{\epsilon}{\epsilon - 1}W &= \frac{\epsilon}{\epsilon - 1}(\frac{W}{P})P. \\ &= \frac{\epsilon}{\epsilon - 1}(\frac{M_1}{P})^{1/\delta}P.\end{aligned}$$

And the profits become,

$$\pi_i^A = \frac{P_i y_i - W L_i}{P} - c.$$

Substituting the price into the profit function gives,

$$\pi_i^A = \left[(\epsilon - 1)^{\epsilon - 1}\epsilon^{-\epsilon}(\frac{M_1}{P})^{(1+\delta-\epsilon)/\delta}\right]/n - c.$$

– Thus if $\pi^A > \pi^F$, then the firm should adjust its prices. If the menu costs, c, are non-trivial, it is more likely the firms will not change prices. Note that the adjustment decision depends on δ which is a coefficient in the labor supply equation. If δ is large, the labor supply curve is rather flat so the wages don't change by much when the shock hits. Therefore, firms will have less to gain from lowering prices. If δ is small however, the labor supply curve is steep and wages will fall by a large amount. Thus adjustment becomes more likely.

- Note that we have shown here that a crucial feature in generating large effects from small menu costs is the presence of significant REAL rigidities elsewhere in the economy (elastic labor supply for example). **[G-9.2]**

9.2 Aggregation in the Menu Cost Models

- With reference to the graph in the notes [**G-9.3**], we have the same setup as before with a firm in an imperfectly competitive market where wages fall from w_1 to w_2 and finally to w_3. Each time they fall, there are extra profits that a firm is forgoing by not lowering its prices in line with the new optimal conditions. However, if there are costs associated with changes prices (menu costs), a firm may choose to stay at the high prices. However, say the costs are too high when wages fall from w_1 to w_2 so firms keeps prices the same, but when they fall again, the extra profits do provide the required incentive and prices are changed from p_1 to p_3, a rather large fall.

- This process results in a sort of "Price/Wage Jerkiness." It is a clear explanation for the nominal rigidities seen in the market. There are no smooth adjustments but rather a sort of jumpiness of wages and prices. [**G-9.3**]

- Even if we consider a dynamic world where nomimal wages change continuously and the profit maximizing price is a constant markup above that wage, we'll still get a steplike pattern of prices rising quickly over one period, but staying constant over another.

- Algebraically, this can be seen as a firm trying to minimize a Loss function of the following form:
$$L = \int_{t=1}^{T} \alpha(P_{it} - P_{it}^*)^2 dt + nc.$$
Where P_{it} is firm i's actual current price at time t, P_{it}^* is the profit maximizing price level for firm i at time t. There is an additional cost of nc as firms incurr a cost, c, for each of the n times that they change prices.

- If an economy is one that has more or less continuous inflation, we would expect less inertia but when there is more uncertainty about prices and wages next period, we would get more jumpiness or jerkiness. Thus a high inflation economy is less sensitive to nominal shocks. Or at least it should be!

9.3 Labor Markets

- We will now consider Implicit Contracts which will show the sources of wage inflexibilities. In this section we are concerned with the "real" or structural theories which attempt to explain real wage rigidity and the possibility of a sustained non market-clearing equilibrium in the labor market. People are assumed to be risk averse so they would prefer stability of wages which make up a major part of their income. However, they cannot hedge their risk by finding several different jobs like one can in a financial market. Human capital is indivisible (except for Ed's).

- Consider identical firms that wish to maximize expected profits, $E[\pi]$, subject to a production constraint, $y = f(L)$, where labor is the only input. Assume $f' > 0$ and $f'' < 0$ as normal.

- The market price for the good to be produced is one of two values, P_1 or P_2 with $P_2 > P_1$, corresponding to two different states of the world. The probability of being in state 1 is $\frac{1}{2}$, the same as state 2.
- Prior to everything, the firm choose how many workers it will hire. A firm will choose between paying a laborer, $w = w_1$ or $w = w_2$, again corresponding to the different states of the world. If the firm fires the worker, he becomes unemployed and cannot find work elsewhere. Thus the worker is sort of stuck in the job with no alternatives and the firms know this.
- Firms specify a contract with wages and also a strategy for laying off people in the bad state of the world (state 1). Let ρ be the employment rate. If we are in state 2, $\rho = 1$ and $\rho L = L \equiv$ full employment. If we are in state 1, the bad state, employment equals ρL with $\rho < 1$.
- A firm maximizes expected profits by determining w_1, w_2, ρ, and L.
- Workers get expected utility equal to:

$$E[u] = \underbrace{\frac{1}{2}u(w_2)}_{State\ 2} + \underbrace{\frac{1}{2}[\rho u(w_1) + (1-\rho)u(\tilde{w})]}_{State\ 1}.$$

  Where $\tilde{w}$ is an unemployment benefit.
- Note that this expected utility must be greater than the utility that the worker could be getting from just being unemployed and making $\tilde{w}$. Thus,

$$E[u] \geq u(\tilde{w}) = \tilde{u}.$$

- The firm maximizes expected profits as follows:

$$E[\pi] = \underbrace{\frac{1}{2}[P_2 f(L) - w_2 L]}_{State\ 2} + \underbrace{\frac{1}{2}[P_1 f(\rho L) - w_1 \rho L]}_{State\ 1}.$$

- To solve this problem, set up the Lagrangian as follows,

$$L = E[\pi] + \lambda(E[u] - \tilde{u}).$$

  Therefore,

$$L = \frac{1}{2}[P_2 f(L) - w_2 L] + \frac{1}{2}[P_1 f(\rho L) - w_1 \rho L] - \lambda(\frac{1}{2}u(w_2) + \frac{1}{2}[\rho u(w_1) + (1-\rho)u(\bar{w})] - \tilde{u}).$$

  Yields FOCs,

$$\frac{\partial L}{\partial w_1} = 0 \Rightarrow \frac{1}{2}[-\rho L + \lambda \rho u'(w_1)] = 0.$$
$$\frac{\partial L}{\partial w_2} = 0 \Rightarrow \frac{1}{2}[-L + \lambda u'(w_2)] = 0.$$

- Thus, solving these two equations simultaneously, we get,

$$u'(w_1) = u'(w_2).$$

  Or,

$$w_1 = w_2.$$

- Thus the optimal contract is one in which the same wage is paid in both states. The workers are very happy with this because it minimizes the variability of their wage and therefore their income.

10 Week 10

10.1 More on Implicit Contracts

- Recall from last week the expected profit and utility functions:

$$E[\pi] = \underbrace{\frac{1}{2}[P_2 f(L) - w_2 L]}_{State\ 2} + \underbrace{\frac{1}{2}[P_1 f(\rho L) - w_1 \rho L]}_{State\ 1}.$$

$$E[u] = \underbrace{\frac{1}{2}u(w_2)}_{State\ 2} + \underbrace{\frac{1}{2}[\rho u(w_1) + (1-\rho)u(\tilde{w})]}_{State\ 1}.$$

To solve the firm's maximization problem, we set up the lagrangian as:

$$L = E[\pi] + \lambda(E[u] - \tilde{u}).$$

Substituting,

$$L = \frac{1}{2}[P_2 f(L) - w_2 L] + \frac{1}{2}[P_1 f(\rho L) - w_1 \rho L] - \lambda(\frac{1}{2}u(w_2) + \frac{1}{2}[\rho u(w_1) + (1-\rho)u(\bar{w})] - \tilde{u}).$$

Yields FOCs,

$$\frac{\partial L}{\partial w_1} = 0 \Rightarrow \frac{1}{2}[-\rho L + \lambda \rho u'(w_1)] = 0.$$

$$\frac{\partial L}{\partial w_2} = 0 \Rightarrow \frac{1}{2}[-L + \lambda u'(w_2)] = 0.$$

- And solving these two equations, we find that $w_1 = w_2 = \bar{w}$. The optimal contract involves setting the same wage in both states of the world (or states of demand). This insulates workers from demand fluctuations.

- Now that we have considered the wage, we would like to figure out what the employment level will be.

- **[G-10.1]** Consider the graph in the notes which shows that if we could set spot wage rates at different times depending on demand, we would set wages higher in the good state and of course lower when demand was low. However, under the optimal contract we set the wage, $\bar{w}$, equal to the expected value of the marginal product of labor.

- The one point that needs to be made however, is the level of unemployment benefits. They present a worker with an alternative to working wages, so they will effect the labor decision.

- If unemployment benefits, $\tilde{w}$, are fairly low, there is no effect on the model and we still get the same spot rates and the same equilibrium contract rate.

- If $\tilde{w}$ is relatively high, ie, close to $\bar{w}$, then the marginal product of labor falls short of the wage in the bad state of the world. **[G-10.2]** Thus paying L workers is inefficient and employment falls to ρL. How does the lagrange change with ρ? Consider,

$$\frac{\partial L}{\partial \rho} = \frac{1}{2}(p_1 f'(\rho L)L - wL) + \frac{1}{2}\lambda(u(w) - u(\tilde{w})).$$

  From above we know that $\lambda = \dfrac{L}{u'(w)}$.

  Thus substituting in and solving,

$$w - p_1 f'(\rho L) = \frac{u(w) - u(\tilde{w})}{u'(w)}.$$

  Since the right side is greater than 0, this implies that,

$$w > MP(L).$$

  And from convexity properties of utility (risk aversion), we know that $MP(L) < \tilde{w}$ in the bad state.

- Thus in the good state of the world, we get to full employment at c_2, but in the bad state of the world, we only employ $c_1 = \rho L$ workers. Wage stability is bought with increased unemployment.

- Though the model leads to wage rigidity, it has less variability of employment than with spot markets. Hence this type of wage rigidity cannot help to explain excessive employment variability.

10.2 Efficiency Wage Models

- While almost all goods are available to anyone prepared to pay the price, all jobs are not available to anyone prepared to work for a given wage.

- Fundamental Assumption: workers will produce more output if they are paid higher wages. Firms can increase profits by paying above the market wage.

- Reasons for efficiency wages:

  - Higher nutritional standards (paid for with higher wages) lead to a more productive workforce (in developing countries).
  - High relative wages make recruitment easier and discourage quits.
  - Improve the average quality of the workforce through self-selection.

- Raises the morale of the workers because they feel as if they are being treated well.
- Prevents shirking.

- The model: Let $L =$ the employment level.
- Let $e = e(\frac{w}{\bar{w}}, u)$ be the effort level by workers as a function of the unemployment rate, u, and their relative wages compared with the outside wage, $\bar{w}$.
- Output per firm $= f(eL)$.
- Two states of the world: 1 and 2 with $e_1, e_2 > 0$.
- Firms maximize

$$\pi = pf(eL) - wL.$$

Substituting,

$$\pi = pf(e(\frac{w}{\bar{w}}, u)L) - wL.$$

FOC(w):

$$\frac{\partial \pi}{\partial w} = pf'(eL)e_1\frac{L}{\bar{w}} - L = 0.$$

FOC(L):

$$\frac{\partial \pi}{\partial L} = pf'(eL)e - w = 0.$$

Solving the first FOC,

$$e_1\frac{L}{\bar{w}} = \frac{L}{pf'(eL)}.$$

$$\frac{e_1}{\bar{w}} = \frac{1}{pf'(eL)}.$$

Solving the second FOC,

$$pf'(eL)e - w = 0.$$

$$\frac{e}{w} = \frac{1}{pf'(eL)}.$$

Setting the two FOC's equal,

$$\frac{e}{w} = \frac{e_1}{\bar{w}}.$$

Or,

$$\frac{we_1}{\bar{w}e} = 1.$$

- This is called the Solow Condition.

- If all firms are the same, in a symmetric equilibrium we must have $w = \bar{w}$ and in consequence,

$$\frac{we_1}{\bar{w}e} = 1.$$
$$\frac{\bar{w}e_1}{\bar{w}e} = 1.$$
$$\frac{e_1}{e} = 1.$$
$$e_1 = e.$$

And since $w = \bar{w}$, $\frac{w}{\bar{w}} = 1$, so,

$$e_1(1, u) = e(1, u).$$

Or

$$\frac{e_1(1, u)}{e(1, u)} = 1.$$

- If, at full employment, $e_1(1, 0) > e(1, 0)$, there is an incentive for firms to bid up wages to increase output, and wages rise and employment falls. Unemployment rises until higher unemployment so depresses outside opportunities that it provides a sufficient spur to efficiency and firms no longer wish to raise their wages.

10.3 Jackman's Last Lecture: The Shapiro-Stiglitz "Shirking" Model

- The only punishment for shirking is to fire the employee so the only incentive for making sure that your employees don't slack off is the increased risk of losing their job. If unemployment benefits are low or the probability of finding a new job is low, this makes shirking more costly.
- Define the effort level e as,

$$e = \bar{e} \text{ if Worker Works.}$$
$$e = 0 \text{ if Worker Shirks.}$$

- Define the utility level of the worker as $w - e$, or,

$$u = w - \bar{e} \text{ if Worker Works.}$$
$$e = w \text{ if Worker Shirks.}$$

- The output of the firm is a function of the L identical workers that they hire: $y = f(L)$. If the workers shirk, $y = 0$.
- Define s as the probablity that a currently employed worker loses his job.
- Define h as the probablity that an unemployed worker finds a job.
- Define q as the probablity that a shirking worker gets caught slacking and gets "SACKED."

- Thus a working worker gets utility $w - \bar{e}$ and has an $(1-s)$ chance of staying with the firm and a s chance of losing his job.

- A shirking worker gets utility w and has a $1-s-q$ chance of staying with the firm and a $s+q$ chance of losing his job.

- Next we need to define value functions for the individuals that are 1.) working hard, 2.) working and shirking, and 3.) being out on the street unemployed. Define the value of being in work and working hard as V_w:

$$V_w = \underbrace{\frac{w-\bar{e}}{1+r}}_{Current\ Utility} + \underbrace{\frac{(1-s)V_w}{1+r}}_{Expected\ Value\ of\ Working} + \underbrace{\frac{sV_u}{1+r}}_{Unemployed\ Value}.$$

  Where V_u is the value of being unemployed.

- Define the value of working and shirking as V_s:

$$V_s = \underbrace{\frac{w}{1+r}}_{Current\ Utility} + \underbrace{\frac{(1-s-q)V_s}{1+r}}_{Expected\ Value\ of\ Working} + \underbrace{\frac{(s+q)V_u}{1+r}}_{Unemployed\ Value}.$$

- Define the value of being unemployed as V_u:

$$V_u = \underbrace{\frac{\tilde{w}}{1+r}}_{Current\ Utility} + \underbrace{\frac{hV_w}{1+r}}_{Expected\ Value\ of\ Working} + \underbrace{\frac{(1-h)V_u}{1+r}}_{Unemployed\ Value}.$$

  Note that this is assuming that firm's are going to pay a high enough wage to ensure non-shirking behavior so if the unemployed guy gets hired, he isn't going to slack off.

- Firms require $V_w > V_s$ to ensure high effort. Thus,

$$w > \underbrace{rV_u}_{Value\ of\ Unemployed} + \underbrace{\bar{e}}_{Effort\ Required} + \underbrace{(r+s)\frac{\bar{e}}{q}}_{Non-Shirking\ Condition}.$$

- Substuting in V_u, we get,

$$w > \underbrace{\tilde{w}}_{Value\ of\ Unemployed} + \underbrace{\bar{e}}_{Effort\ Required} + \underbrace{\frac{(r+s+h)\bar{e}}{q}}_{Non-Shirking\ Condition}.$$

- In the steady state, the amount of people leaving unemployment is the same as the number of people getting fired. Thus,

$$sL = hU.$$

- The Unemployment rate is defined as $u = \frac{U}{L+U}$. Multiplying everything by s,

$$u = \frac{sU}{sL + sU} = \frac{sU}{hU + sU} = \frac{s}{s+h}.$$

- Thus $s + h = \frac{s}{u}$. So,

$$w > \underbrace{\tilde{w}}_{Value\ of\ Unemployed} + \underbrace{\bar{e}}_{Effort\ Required} + \underbrace{\frac{(r + \frac{s}{u})\bar{e}}{q}}_{Non-Shirking\ Condition}.$$

  Overall, this is the Non-Shirking condition [**G-10.3**] that firms need to determine the wage they need to pay to make sure workers are providing high effort. Note that if q is high, or the probability that a shirking workers gets detected is high, then the firm does not have to pay such a high premium to make sure the workers are giving high effort. Also if $u \to 0$, then $w \to \infty$, so when there is no unemployment, the non shirking wage goes to infinity. This is intuitive because there is no punishment for losing a job, as one could immediately find a new job if detected shirking.

- Setting the non-shirking condition equal to the demand equation, $w = f'(L)$, we get the equilibium wage and employment level which includes some wage premium above $\tilde{w} + \bar{e}$ and a certain amount of unemployment to provide incentives.

- Note that in this last step of aggregating over all firms to determine the macro effects of a non-shirking condition is a bit shakey. All firms are surely not identical and have different monitoring costs so this may not be very likely in real life. So the relevance of efficiency wages to the Macro economy is weak.

- To sum up, considering the recent current events with a weak economy and the WTC disaster, the economy has taken a hit. IS has shifted way back as investment has fallen off and the technology bubble has burst. Thus to keep things in line the central banks have been instrumental in cutting interest rates to avoid output falling too far. Considering the AD/AS model, since AD has shifted way in, if prices are perfectly flexible, this shift in AD would simply cause the price level to fall and we would maintain full employment. However, as has been shown for various reason (Menu Costs etc), prices are NOT perfectly flexible so the interference of the central banks to cut rates, stimulate investment, and push AD back out is essential to keeping employment and output up. Otherwise output would fall with AD at the current price level which could be disasterous.