The Effects of Minimum Wage Policy on Employment and Poverty Levels

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1 Introduction

Since 1938, the minimum wage has been used as a tool for reducing income inequality and diminishing the poverty level in America. A historical chart of the federal minimum wage is plotted in figure 1.[1] An incredible amount of analysis has been done, both empirical and theoretical, to determine if setting a wage floor in an economy accomplishes these goals. Two important works, one by David Card and Alan Krueger [4], and another by John Addison and McKinley Blackburn [1], find that increasing the minimum wage leads to higher levels of employment in the economy, lower levels of poverty, and overall makes the economy better off. Contrary to these studies, there are many articles that show just the opposite. This paper presents several of these findings and then draws on two models to aid in the understanding of the results. The following analysis will proceed with a general review of the evidence on the effects of the minimum wage. In section II and III, the monopsony and efficiency wage model will be considered. Both have been cited as possible explanations for some of the counter-intuitive findings that will be discussed. Finally in section IV, an extention of the efficiency wage model is analyzed which involves the impact of a change in welfare policy on the model. Though models do exist to show the positive effects of a minimum wage, most economists agree that it must be used in combination with other work incentive policies, such as the Earned Income Tax Credit (EITC), to combat unemployment and poverty in America.

A fairly simple concept obtained from introductory economics courses states that increasing the price of an input will cause a firm to reduce the quantity demanded of that input. Consider the labor market graph in figure 2. A competitive firm always sets MRP_L (Marginal Revenue Product of Labor) equal to MC_L (Marginal Cost of Labor). This is because the firm wants to hire workers just up until the point where the additional revenue gained from that employee equals the amount it costs the firm to hire him. The wage, w_*, is set in the market which is in the left panel. This translates into a horizontal MC_L curve for the firm at the wage level. Hence there is a competitive market equilibrium at L_f* laborers and an equilibrium wage of w_*. Now consider

[1] Refer to Appendix A for all referenced figures.

instating a minimum wage floor of w_{min}. The MC_L curve for the firm now shifts up to the new minimum wage. Thus the market equilibrium would now be L'_m workers and a wage of w_{min}. The new level of firm employment would be L'_f. As expected, the minimum wage floor causes the level of employment in each firm and in the economy to fall. However since the equilibrium wage rises, the question now becomes, what is the effect on average earnings and what segment of the population is losing jobs? The usual response is that average earnings rise because the employment effect is fairly small and that many workers from the lowest paid segment of the population become unemployed. However, the evidence of this is varied.

2 Review of Evidence

Addison and Blackburn [1] outline their analysis of how an increase in the minimum age reduces the poverty level, specifically among teenagers and junior high school dropouts. Measuring the impact of minimum wage policy on poverty is a relatively daunting task. Indeed, the methodology of measuring the poverty level alone is subject to much criticism. The most common problem is that the minimum wage has almost no effect on 98.2% of the work force because their wage is already above the minimum wage or they are not covered[2] by it. [2] Secondly, many minimum wage workers are teenagers that come from families that taken as a whole are not below, or even near, the poverty level. Thus Addison and Blackburn analyze state level minimum wage policy using panel data from families with low wage workers and then they measure the effects over a long period of time to attempt to capture the complete effects of the wage floor increase. After regressing the poverty rate on wages, state specific effects, and other fixed effects, they find the poverty/wage elasticities for the 1990's as shown in table 1. Thus among teenagers and junior high dropouts, a 10% increase in the minimum wage will result in a 5% decrease in the poverty level among these two groups. [1] However, when they studied the 1980's they found that there was no statistically significant effect on the reduction in the poverty level following a minimum wage increase. This

[2] 49% of minimum wage workers are employed in retail, where tips and commissions add to their salaries.

Table 1: Wage - Poverty Elasticities

Population Segment	Poverty / Wage Elasticity
Teenagers	-0.50
Young Adults	-0.28
Junior High Dropouts	-0.50
3 Groups Combined	-0.36

could possibly be due to the extremely tight labor markets of the 1990's. If unemployment is very low in an economy, the demand for labor becomes very inelastic. Therefore, a 10% increase in the minimum wage will cause a less than 10% decrease in employment. Thus average earnings will rise and it is more plausible to see a decrease in the poverty rate. This is one explanation for why they concluded the effectiveness of minimum wage policy in the 1990's but not in the 1980's.

Bruce Bartlett estimates the overall wage/employment elasticity to be about -0.21%. [2] This means that a 10% increase in the minimum wage results in a 2.1% decline in the employment rate. However, only 21.3% of workers would be affected by the minimum wage hike so the real absolute elasticity is relatively larger.[3] Aside from employment losses, the minimum wage can also have several other negative impacts, including:

- Increased crime due to higher unemployment.
- Increased employment of illegal aliens willing to work at sub-minimum wage levels who are less likely to report the violation.
- Increased welfare dependency.
- Marginal firms closing their doors and fewer firms starting up.
- Firms cutting back on worker hours, training, and benefits to cover the cost of the wage hike.

[3] A Worker is affected if he is making at or just above the minimum wage

Table 2: Minimum Wage Impact by Income Decile

Decile	% Whose Earning Rise
Poorest	3.5%
2nd	6.6%
3rd	7.4%
4th	8.4%
5th	9.0%
6th	9.5%
7th	7.7%
8th	6.7%
9th	3.1%
Richest	1.9%

Source: Institute of Fiscal Studies, UK

To determine the minimum wage effects on the overall poverty level, further analysis must be done. However, just looking at a few numbers from the Bureau of Labor Statistics, the effects do not look positive. The data shows that 50% of the minimum wage benefits are going to families that are already making at least three times the poverty level! Consider table 2, which measures the impact of the minimum wage on UK household earnings by income decile. The effects are obviously highest for middle income families. The interesting part is that only 3.5% of UK families in the lowest 10% of the income distribution gain from instating a minimum wage. [5]

In the US, David Neumark and William Wascher have attempted to determine the precise effects of minimum wage policy on poverty levels. In an important article they wrote along with Mark Schweitzer[6], they ran regressions of wages, worker hours, employment, and earnings on the minimum wage and other fixed effects. They included variables to account for the lagged effects of

Table 3: Probabilities of Making the Transition Into and Out of Poverty

		Wage Increase		No Increase		Difference	
	Year 1	Poor	Non-Poor	Poor	Non-Poor	Poor	Non-Poor
Year 2							
Poor		0.655	0.066	0.634	0.062	0.022	0.004

each one of these dependent variables. Though wages initially rise following a minimum wage hike, a significant part of that increase is "given back" in the second year. Perhaps by employers taking advantage of inflationary effects. Employment effects are hardly significant and mixed across the distribution. They find that the coefficient on earnings is initially positive, but by the second year it is strongly negative. For example, for those people earning within ten cents of the minimum wage, a 10% increase in the minimum wage results in a 14.7% decrease in their earnings by the second year. A statistic that is significant at the 1.0% level. Thus, including the lagged variables more accurately captures the real effects of a minimum wage increase. This also might be a reason for Card and Krueger's unlikely results in their one year fast food industry study.

Neumark and Wascher also attempted a slightly different type of analysis in which they try to measure the impact of the minimum age on the probability of being poor.[7] This eliminates some of the bias concerning which population segments are affected and if firms are responding by cutting back on hours or fringe benefits. Their findings are summarized in table 3. After accounting for lagged effects, 65.5% of families that are poor in year 1 will remain poor in year 2 following a minimum wage increase. In states that did not have a minimum wage increase, 63.4% of poor families in year 1 remain poor in year 2. The difference between the two estimates is significant at the 5.0% level and indicates the adverse effects of minimum wage policy on poverty levels. Also, 6.6% of those non-poor families in year 1 became poor in year 2 with a minimum wage increase, while 6.2% of non-poor famililes in year 1 became poor in year 2 without a minimum wage increase. A difference that is significant at the 5.0% level. Thus a minimum wage hike increases the probability that poor families remain poor by 2.2% and increases the probability that non-poor families become poor by 0.4%. Both of these estimates are also significant at the 5.0% level. However, after accounting for several controls, such as, AFDC benefits and state and year effects, an increase in the minimum wage reduces the probability that someone poor in year 1 remains poor in year 2 by 8.5.% It also increases the probability that someone comes out of poverty in year 2 by 2.4%.[4]

We will now turn to the study done by Card and Krueger in 1992.[4] They did a comparative analysis of the fast food industry in two adjacent areas of Pennsylvania and New Jersey. The minimum wage in New Jersey rose during the study while in Pennsylvania, the wages remained constant. They found that as a result of the increase, employment in New Jersey rose by 0.59 full-time-equivalent employees (FTE's), while it fell in Pennsylvania by 2.16 FTE's. Furthermore, they found that among workers in the New Jersey stores that were making above the minimum wage, there was a similiar reduction in employment (-2.04 FTE's). Those that were making the minimum wage accounted for the overall increase (+1.32 FTE's). This study has come under considerable criticism for many reasons and most consider it a good example of how *not* to do economic analysis. However, combined with the Addison and Blackburn study, it is worth considering if there is a way to model some of these results.

3 The Monopsony Model

One of the standard models that explains positive employment effects given an increasing wage is the monopsony model. A firm has monopsony power if they are the only, or one of few, buyers of labor in a market.[5] Unlike a competitive market for labor (See figure 2), where firms take the wage level as fixed, a monopsonist affects the equilibrium wage based on how many workers they hire. Consider the monopsony model in figure 3.

[4] Only the latter of these two measures is significant at the 5% level.

[5] The US department of Defense is often described as a monopsonist in the purchase of weapon systems. The Federal Aviation Administration is a monopsony in hiring air traffic controllers.

As always, firms choose their equilibrium wage and quantity of laborers by setting $MRP_L = MC_L$. This intersection determines L_*. For the firm to hire an additional worker, they must raise the wage to attract more workers. However, they must raise the wage not only for that worker, but for all those already employed. Thus MC_L is upward sloping and above S_L. Based on L_*, the monopsonist does not have to pay a wage equal to $MC_L(L_*)$. Instead, they only have to pay a wage w_*, which is the wage corresponding to $S(L_*)$. Once a minimum wage is instated, $MC_L = w_{min}$ up until the intersection with S_L. We reach a higher level of employment at L'.

The problem with a monopsony explanation for the above findings is that a monopsony is very rare. Hardly ever will one find a market where one firm is the sole buyer of labor. The restaurants in the fast food industry that Card and Krueger analyzed might have had some monopsony power due to being the largest employer of low-wage workers, but they would not have been a true monopsony. In order for a wage increase to cause an employment gain, the price of the output product must rise to cover the cost. Since this did not happen in the New Jersey restaurants, the monopsony model solution is not sound.

4 The Rebitzer and Taylor Efficiency Wage Model

Since the monopsony model proved to be an unlikely explanation for the results described thus far, economists have looked to other models. Bhaskar and To considered the monopsonistic competition model where firms have monopsony power due to non-wage differentials between jobs.[3] In other words, since different people prefer different types of occupations, a certain amount of monopsony power is possible even for a small firm that employs from a large labor market. Rebitzer and Tayor consider the efficiency wage model to explain some of these findings.[8] In standard labor market analysis, the wage is normally set by the market forces. In the efficiency wage model, the wage is set above the market wage due to the added benefits of paying a higher wage. It is based on the assumption that the productivity of the workers hired in a firm is a function of their wage. Paying the efficiency wage means that the firm will attract more productive workers, have less job turnover, and lower recruiting costs. The intuition behind the following analysis is as follows,

Higher Wage $\Rightarrow$ Higher Cost of Losing Job $\Rightarrow$ Lower Monitoring Cost $\Rightarrow$ Increase Employment.

To begin this model, consider a firm that maximizes a standard profit function, $\pi(w,l)$.[6] To accomplish this, the firm determines optimal labor and wage by taking the partial derivatives of π with respect to l and w and setting them equal to zero. Rearranging the terms of these optimizations yields the following equation:

$$\frac{dl}{dw} = \frac{-\partial^2\pi/\partial w\partial l}{\partial^2\pi/\partial l^2}. \tag{1}$$

This derivative will normally have a negative sign, indicating that an increase in the wage will have to be accompanied by a decrease in employment. Thus $dl/dw < 0$. We will show that in some cases (besides monopsony) the sign on this derivative could be positive, thus explaining the positive employment effects of a minimum wage.

We will start with the assumption that there are a large number of small firms and each of these firms must monitor their employees to make sure they are being productive and not shirking. The cost of monitoring the employees increases with the size of their workforce. Define an output function for the firm, $f(w,l) = g(l)$ if and only if workers are not shirking. Otherwise, $f(w,l) = 0$, because a firm cannot be productive if its employees are not putting a sufficient amount of effort, e, into their jobs. Now, consider a worker at this firm. He or she must decide to work and provide an effort level of at least e, or to shirk, but risk getting caught and dismissed from the job. The decision would surely not only depend on the wage of the present job, but also the utility he would gain from working elsewhere. To continue the analysis, we must first define several new variables.

V^N = The level of utility gained from NOT shirking.
V^S = The level of utility gained from shirking.
V^A = The level of utility gained from the next best alternative.
$\bar{w}$ = The level of utility gained from not working.

The precise definitions of these utility equations can be found in Appendix B. These three equations make intuitive sense once one breaks them down into several parts. The expected flow

[6] Additional equations and computations are available in Appendix B.

of utility from not shirking, V^N, is first a function of the wage minus the effort that must be put into the job. It is also a function of the recursive term which accounts for the utility from working in the future, given the possibility of quitting and discounted by $(1+r)$. The last term comes from the possibility that a worker will quit the firm and resort to the next best alternative. The expected lifetime utility of a shirker, V^S, is derived from their wage, the same recursive term as above, but this time is reduced by $(1-D)$, or the probability of not getting caught. Finally the last term reflects the utility from leaving the firm, either by getting caught or quitting, and then moving into the next best alternative. The utility gained from persuing an alternative way of life, V^A, is a function of the $\bar{w}$ term, plus a discounted recursive term that reflects the probability of attaining another job versus remaining unemployed.

The goal of the firm, by ensuring that their workers are being productive and not shirking, is to set a wage such that $V^N = V^S$. They want to make sure that the utility that an employee gets from working hard, which is a function of their wage, is just high enough to be greater than the utility from shirking. Setting equation 13 equal to equation 14, a firm determines the lowest possible wage it can pay to assure that its workers provide optimal effort. This reduces to the non-shirking wage, w_{ns}, such that:

$$w_{ns} = \bar{w} + e + \frac{e(r+s+q)}{D(1-q)}. \tag{2}$$

Substituting w_{ns}, the efficiency wage, into the production function, we obtain the profit function, $\pi(w,l) = f(w,l) - wl = g(l) - w_{ns}l$. Taking the derivative of profits with respect to labor, the resulting equation is simply the difference between marginal product and marginal cost since $g(l)$ is equal to total product and $w_{ns}l$ is equal to total cost. This is a normal result one would expect for a profit maximizing firm. To determine $l'(w)$, take the reciprocal of the derivative of equation 2 with respect to l, yielding:

$$l'(w) = -\frac{D^2(1-q)}{e(r+s+q)D'(l)} > 0. \tag{3}$$

Now $l'(w)$, or equivalently, dl/dw, is the relationship that we have been trying to find from the beginning. It is going to be positive because all of its terms are positive. Thus the no shirking condition is upward sloping. In order to show that the profit maximizing condition also holds when wages and labor rise, a graphical representation is necessary. Consider the efficiency wage model in figure 4.

Figure 4 is actually just the competitive labor market model that was shown in figure 2, but also included is the no-shirking condition and an isoprofit curve. The no-shirking curve represents the lowest wage that would guarantee high effort. Notice that the no-shirking curve and MC_L are upward sloping reflecting the increasing cost of monitoring larger numbers of workers and the result in equation 3. The isoprofit curve represents combinations of wages and quantity of labor where profits are equal. The firm continues to set $MRP_L = MC_L$ to determine L_0, but similar to monopsonists, they do not have to pay a wage equal to the marginal revenue product. With the inclusion of the no-shirking constraint, before the minimum wage is in place the firm hires L_0 workers at a wage of w_0 because this is where the isoprofit curve is just tangent to the no shirking condition. Moving away from this point will either result in lower profits if the wage is above w_0 or a violation of the no-shirking requirement if the wage is below w_0. Now, consider a binding wage floor w_{min}. With the wage in place, $MC_L = w_{min}$. Since initially, $MC(L_0)$ was above w_{min}, instating a minimum wage actually lowers the marginal cost of hiring an additional employee. This results in a firm hiring additional workers and employment rises to L_1. As shown in figure 4, since the wage must be high enough to satisfy the no-shirking condition, the firm must now be on a new (lower) isoprofit curve, but they are at *virtually* the same level of profits as before.

After the minimum wage increase, firm profits do fall in the short term. For the long run analysis, we will extend the profit function as follows,

$$\pi = pg(l) - wl - R. \tag{4}$$

Where R represents the normal profit of the firm, or the cost of using one's entrepreneurial abilities in the production of a good. Therefore π is truly the economic profit of the firm. To determine the wage, as was shown before, take the derivative of profits with respect to labor and set it equal to zero. Also set equation 4 equal to zero because in the long run, firms should be making zero economic profit. This simplifies to,

$$p(AP_L - MP_L) = \frac{R}{l} - lw'(l). \tag{5}$$

We find that setting these two equations equal yields an equation involving the difference between the average product of labor and the marginal product of labor. Consider figure 5 which shows their relationship to each other and to marginal and average cost.

We will now consider several cases in the long run.

- If R is small, the right hand side of equation 5 will be negative implying that $MP_L > AP_L$. This implies that the firm is on the downward sloping portion of their long-run average total cost curve. It has been shown that following a minimum wage increase, employment will rise in the short run. Once profits fall, firms will be forced to leave the industry, causing total employment to fall back to the original level. However, at the remaining firms in the market, employment per firm has increased. Since the firms are on an increasing portion of their AP_L curves, industry output will have risen if only a relatively small number of firms exited, and price therefore must fall. If profits are still negative, more firms will exit. However, in the end, though per firm employment has risen, industry employment is lower.

- If R is large, the right hand side of equation 5 will be positive implying that $MP_L < AP_L$. This implies that the firm is on the upward sloping portion of their long-run average total cost curve. So following the minimum wage increase, employment will rise in the short run. Once profits fall, firms will be forced to leave the industry, causing total employment to fall back to the original level. However, at the remaining firms in the market, employment is higher per firm. Since the firms are on a decreasing portion of their AP_L curves, industry output will have fallen, and price therefore must rise. Profit will be positive which will cause firms to enter until the zero profit equilibrium is attained. Thus, industry employment rises above the previous equilibrium level in the long run.

- If $\frac{R}{l} - lw'(l) = 0$, then $MP_L = AP_L$, and firms must be operating at the minimum of their long run average cost curve. An increase in the minimum wage here will cause employment to rise in the short run and profits will fall. As firms exit the industry, industry employment falls back to its original level though per firm employment has actually risen.

This model has clearly shown that via the efficiency wage model, one can show that an increase in the minimum wage will have positive employment effects in the short run and ambiguous effects in the long run. However, if firms are initially operating at the minimum of their long run average cost curve, long run industry employment remains unchanged.

5 Extension of Policy Analysis

To extend this model further, let us now consider a slight change in the assumptions. First assume that the minimum wage is already in place and firms are hiring L_1 workers. The no-shirking requirment was derived from the utilities that a potential employee would get from working hard, from working and shirking, or from pursuing other alternatives. Thus an exogenous change in those other alternatives will have an effect on the model. Consider the welfare policy changes of 1996 that included new limits placed on benefits. This decrease in welfare benefits causes the cost of shirking for an employee to increase because there is a smaller safety net if he loses his job. Therefore, from the point of view of the firm, the no shirking condition has shifted down. The firm must pay a lower wage to guarantee that its employees do not shirk because they have more of an incentive to hold on to their job. This is shown graphically in figure 6 as we move from *No Shirk(1)* to *No Shirk(2)*. This is accompanied by a similiar shift in marginal cost from MC_L to MC_L'.

Firms are now subject to two constraints when determining the number of employees to hire, w_{min} and *No Shirk(2)*. Since they are still profit maximizing, the firm will be able to attain a higher level of profits which is indicated by the shift from *Iso(1)* to *Iso(2)*. The wage remains at w_{min}, but employment has increased to L_2. Thus, not only did an increase in the minimum wage increase employment, but limiting welfare benefits increased employment and firm's profits!

Though employment has increased and the firms are happy making larger profits, what about the welfare of the work force? Consider the following equation,

$$V^A = \bar{w} + \frac{sV^N + (1-s)V^A}{(1+r)}. \tag{6}$$

V^A represents the utility that a person would receive from leaving his present employer and pursuing other alternatives. Since welfare benefits have decreased, the $\bar{w}$ term must be lower than before, representing a lower utility from not working. The variable s, which is the probability of success in finding a new job, has risen with the higher level of employment in the economy. The change in V^A is ambiguous and is dependent on the utility one gains from working productively. One must then pose the question, is society better off overall? The answer seems to be in the trade off between the welfare of those with jobs versus those that still are not fortunate enough to find work. Employment in the economy has risen, and those that are working are getting paid higher wages, but those that are unemployed are getting hurt by lower welfare benefits. Thus the debate over the amount of welfare support continues and the efficiency wage model has shown that the implications of such policies can lead to mixed results.

6 Conclusions

The recent analysis done on the effects of minimum wage policy on employment and poverty levels leads to mixed conclusions. The employment effects appear to be rather small, and the monopsony and efficiency wage models are useful for explaining some of the irregularities that have been found. The minimum wage alone appears to adversely affect the poverty rate. Perhaps some combination of minimum wage policy and the EITC would prove successful in bringing down the level of poverty. Economists must continue to study the effects of minimum wage policy to determine if it is simply a contractionary policy or if it is truly helping to redistribute income from high to low income groups and therefore reduce the poverty level. Considering the analysis done here, it seems unlikely that the minimum wage can be used alone as a tool for creating any significant benefits to those that need them the most.

References

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[3] Bhaskar, V. and Ted To. "Minimum Wages for Ronald McDonald Monopsonies: A Theory of Monopsonistic Competition." *The Economic Journal.* V. 109, 1999.

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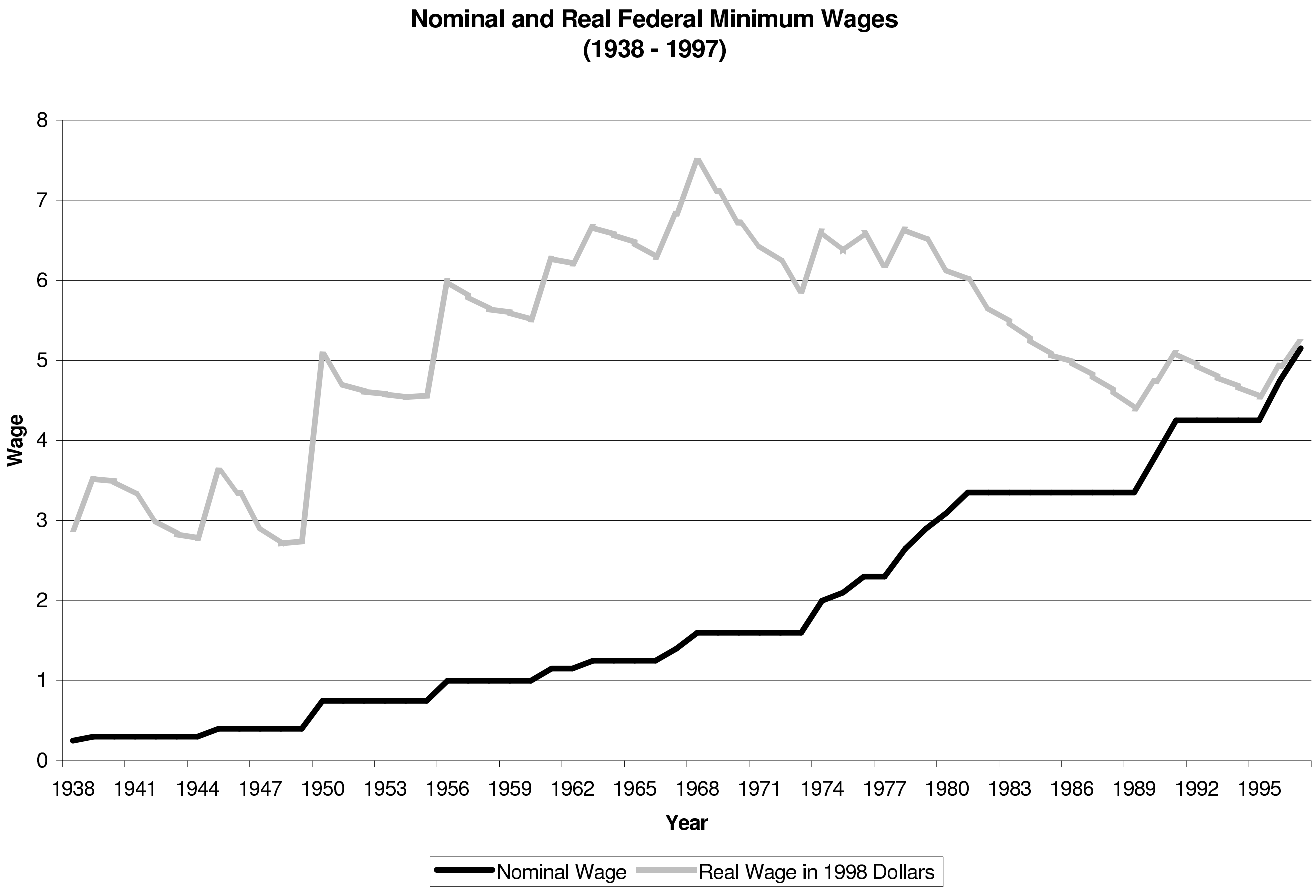
[7] Neumark, David and William Wascher. "Do Minimum Wages Fight Poverty?" NBER working paper 6127, 1997.

[8] Rebitzer, James B. and Lowell J. Taylor. " The Consequences of Minimum Wage Laws - Some New Theoretical Ideas." *Journal of Public Economics.* Vol 56, 1995.

Appendix A - Figures

Figure 1: Historical Minimum Wages



Source: US Department of Labor

Figure 2: Competitive Labor Market Model

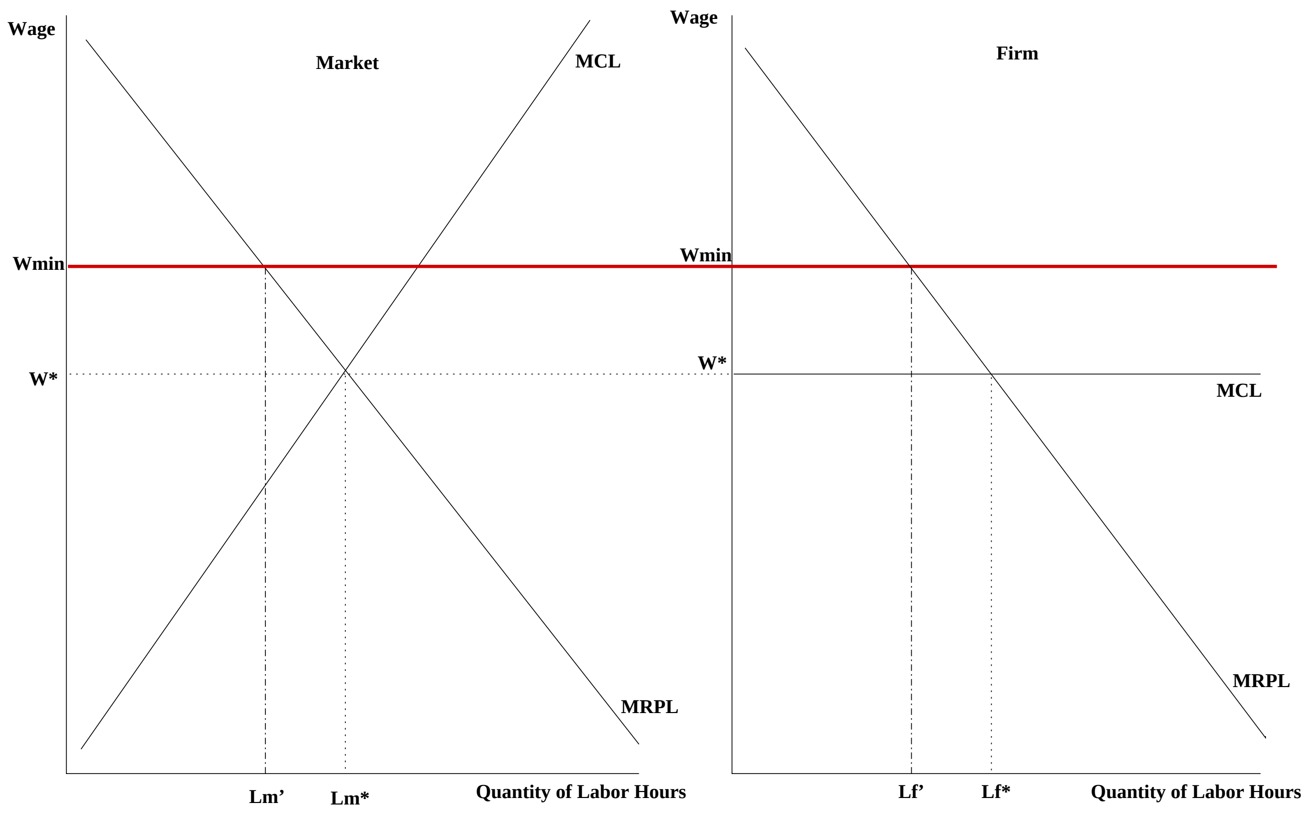


Figure 3: Monopsony Model

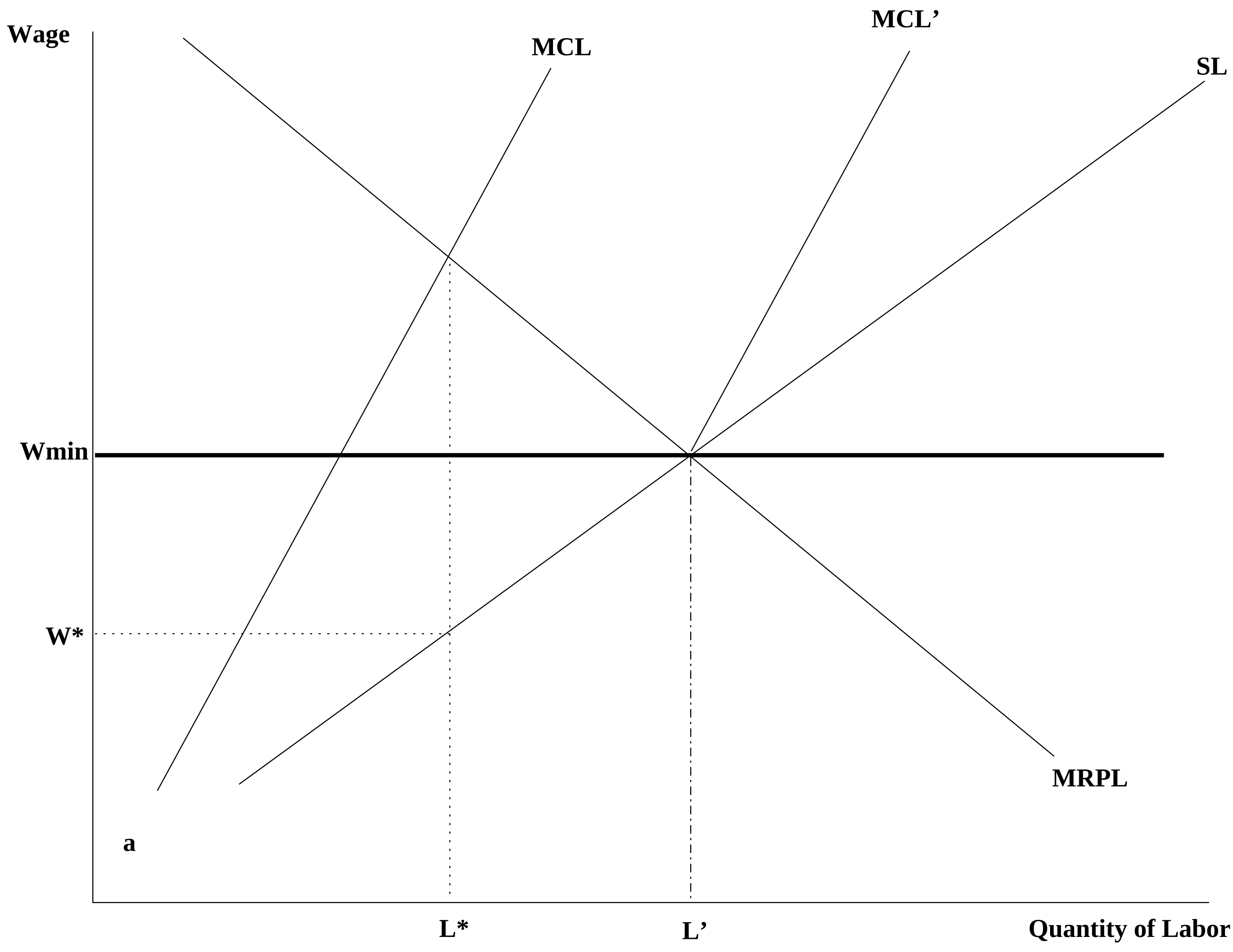


Figure 4: Efficiency Wage Model

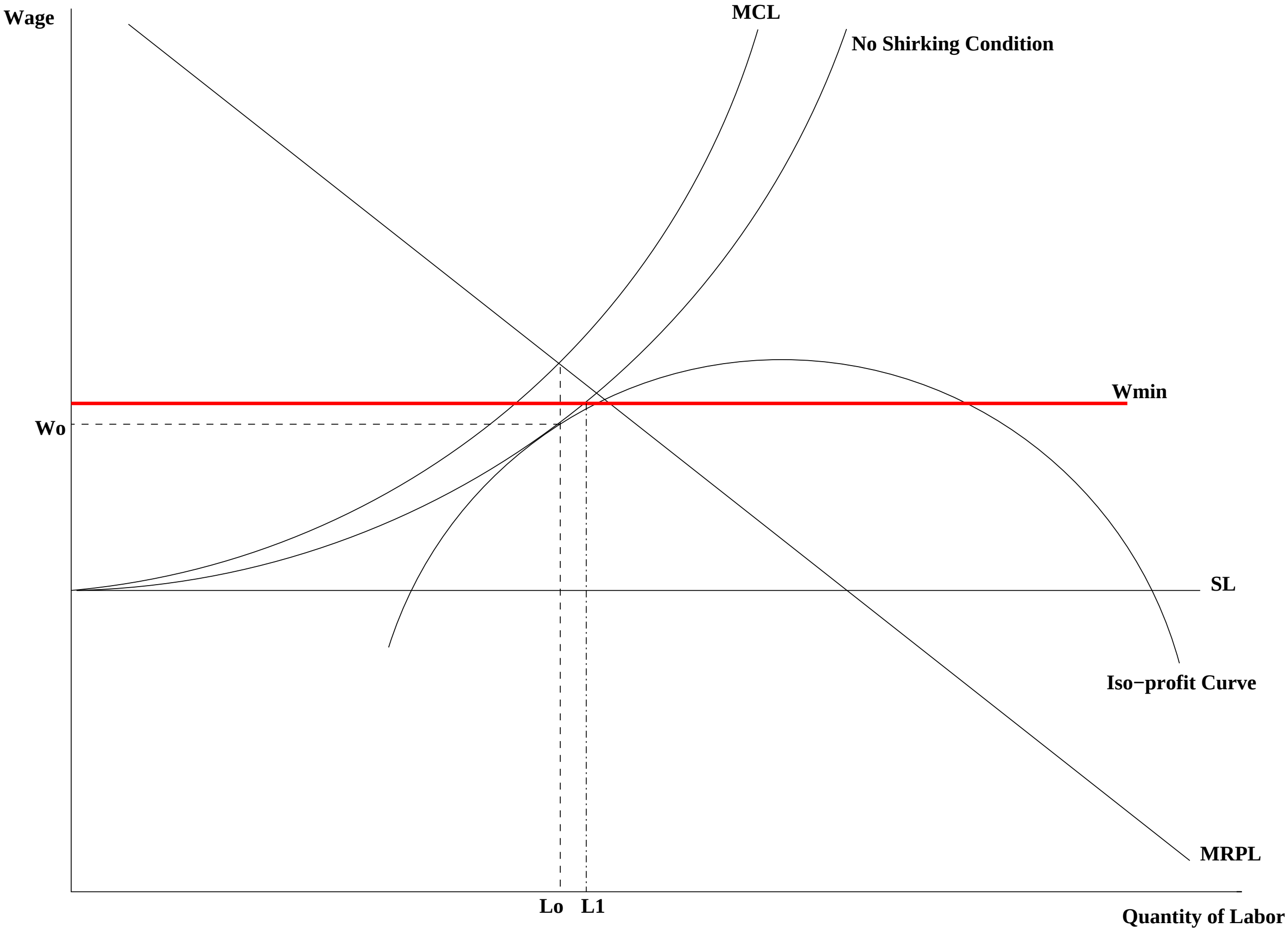


Figure 5: Average and Marginal Product versus Average and Marginal Cost

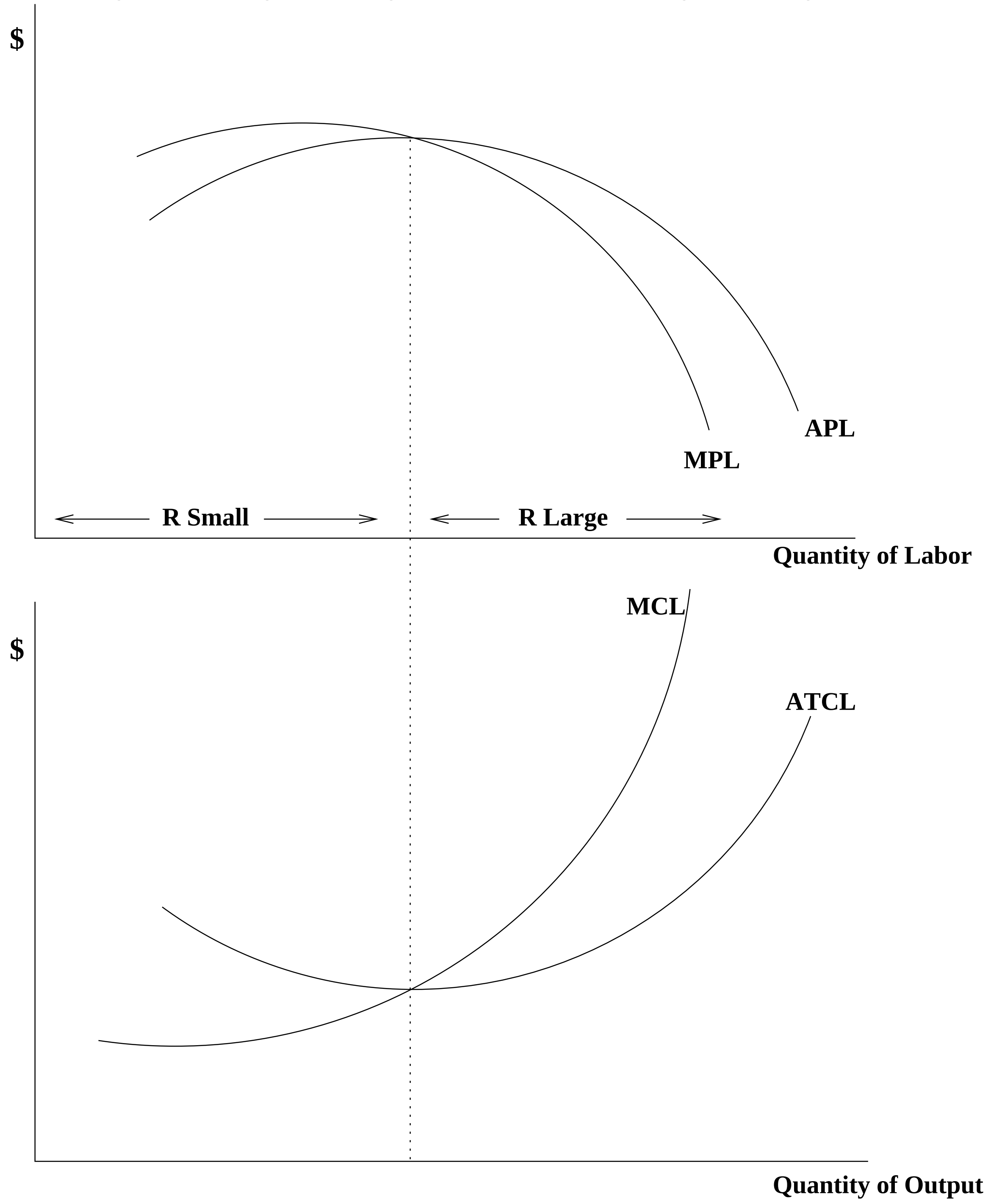
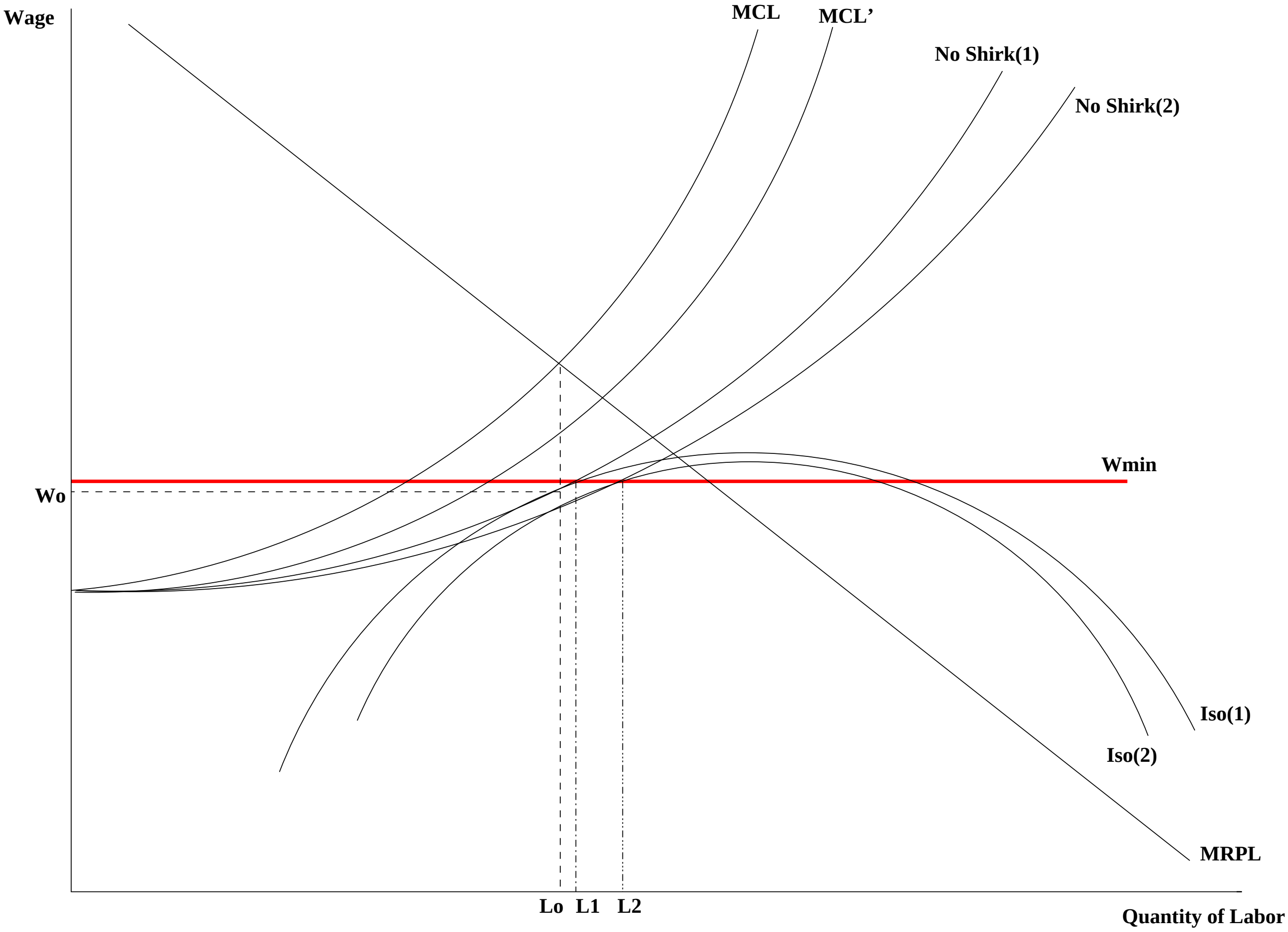


Figure 6: Adjustment to Efficiency Wage Model



Appendix B - Equations and Computations

Included in this appendix is the mathematics behind the equations summarized in Section IV and V of this paper. Those in the text should provide an adequate understanding of the dynamics of the model, but these are included for reference if anything is unclear.

A profit maximizing firm will optimize according to,

$$H = \frac{\partial \pi}{\partial w} = 0, \tag{7}$$

$$K = \frac{\partial \pi}{\partial l} = 0. \tag{8}$$

Hence by the implicit function theorem,

$$\frac{dw}{dl} = \frac{-\partial H/\partial l}{\partial H/\partial w}. \tag{9}$$

Thus, substituting in H,

$$\frac{dw}{dl} = \frac{-\partial^2 \pi/\partial w \partial l}{\partial^2 \pi/\partial w^2}. \tag{10}$$

Similarly, by solving and substituting in K,

$$\frac{dl}{dw} = \frac{-\partial^2 \pi/\partial w \partial l}{\partial^2 \pi/\partial l^2}. \tag{11}$$

The production function can be defined as follows,

$$f(w,l) = \begin{cases} g(l) & \text{if } \epsilon = e \\ 0 & \text{if } \epsilon = 0. \end{cases} \tag{12}$$

Utility variables:

V^N = The level of utility gained from NOT shirking.
V^S = The level of utility gained from shirking.
V^A = The level of utility gained from the next best alternative.
D = The probability that a firm detects an employee who is shirking.
q = The probability that an employee quits the firm.
r = A discount rate.
s = The probability of success in finding another job.
$\bar{w}$ = The level of utility gained from not working.

The equations for V^N, V^S, and V^A, in terms of the other variables defined thus far:

$$V^N = w - e + \frac{(1-q)V^N}{(1+r)} + \frac{qV^A}{(1+r).} \tag{13}$$

$$V^S = w + \frac{(1-q)(1-D)V^S}{(1+r)} + \frac{[1-(1-q)(1-D)]V^A}{(1+r).} \tag{14}$$

$$V^A = \bar{w} + \frac{sV^N + (1-s)V^A}{(1+r)}. \tag{15}$$

The new production function,

$$f(w,l) = \begin{cases} g(l) & \text{if } w \geq w_{ns} \\ 0 & \text{if } w < w_{ns}. \end{cases}$$

Taking the derivative of profits with respect to labor,

$$\frac{d\pi}{dl} = g'(l) - [w + lw'(l)] = 0. \tag{16}$$

We obtain this via the chain rule because the wage is now a function of the amount of labor employed at a firm. This was one of our assumptions: D the probability of detecting a shirker is a function of the size of the labor force. Since D is in our equation for w_{ns}, w_{ns} is a function of l. To determine $w'(l)$, take the derivative of equation 2 with respect to l, yielding:

$$w'(l) = -\frac{e(r+s+q)D'(l)}{D^2(1-q)} > 0. \tag{17}$$

Similiarly,

$$l'(w) = -\frac{D^2(1-q)}{e(r+s+q)D'(l)} > 0. \tag{18}$$

Long Run Analysis.

Optimize profit function such that,

$$\partial\pi/\partial l = pg'(l) - [w + lw'(l)]. \tag{19}$$

Setting this equal to zero and solving for w,

$$w = pg'(l) - lw'(l). \tag{20}$$

Since this is a long run analysis, we can assume the economic profits are 0, or $wl = pg(l) - R$. Thus,

$$w = \frac{pg(l) - R}{l}. \tag{21}$$

Setting the zero profit criteria equal to the optimization criteria,

$$\frac{pg(l)}{l} - \frac{R}{l} = pg'(l) - lw'(l). \tag{22}$$

$$p(\frac{g(l)}{l} - g'(l)) = \frac{R}{l} - lw'(l). \tag{23}$$

$$p(AP_L - MP_L) = \frac{R}{l} - lw'(l). \tag{24}$$